Model Question Paper-1 with effect from 2019-20 (CBCS Scheme)

USN

Fourth Semester B.E. Degree Examination

Engineering Statistics & Linear Algebra

TIME: 03 Hours **Max. Marks: 100**

Note: 01. Answer any **FIVE** full questions, choosing at least **ONE** question from each **MODULE**.
02. Use of Normalized Gaussian Random Variables table is permitted.

<table>
<tr><th colspan="3">Module -1</th><th>*Bloom's Taxonomy Level</th><th>Marks</th></tr>
<tr><td>Q.01</td><td>a</td><td>The PDF for the random variable Z is $$f_Z(z) = \begin{cases} \frac{1}{6\sqrt{z}} & ; \quad 0 < z < 9 \\ 0 & ; \quad Otherwise \end{cases}$$ What are (i) the mean (ii) the mean of the square, and (iii) the variance of the random variable Z?</td><td>L1, L2</td><td>5</td></tr>
<tr><td></td><td>b</td><td>Given the data in the following table,
<table><tr><td>k</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td></tr><tr><td>x_k</td><td>2.1</td><td>3.2</td><td>4.8</td><td>5.4</td><td>6.9</td></tr><tr><td>$P(x_k)$</td><td>0.21</td><td>0.18</td><td>0.20</td><td>0.22</td><td>0.19</td></tr></table>(i) Plot the PDF and the CDF of the discrete random variable X.
(ii) Write expressions for PDF and CDF using unit-delta functions and unit – step functions.</td><td>L3</td><td>5</td></tr>
<tr><td></td><td>c</td><td>Define an exponential random variable. Obtain the characteristic function of an exponential random variable and using the characteristic function derive its mean and variance.</td><td>L1, L3</td><td>10</td></tr>
<tr><td colspan="3">OR</td><td></td><td></td></tr>
<tr><td>Q.02</td><td>a</td><td>It is given that $E[X] = 36.5$ and that $E[X^2] = 1432.3$
(i) Find the standard deviation of X.
(ii) If $Y = 4X - 500$, find the mean and variance of Y.</td><td>L3</td><td>4</td></tr>
<tr><td></td><td>b</td><td>Define a Poisson random variable. Obtain the characteristic function of a Poisson random variable and hence find mean and variance using the characteristic function.</td><td>L1, L3</td><td>10</td></tr>
<tr><td></td><td>c</td><td>The random variable X is uniformly distributed between 0 and 4. The random variable Y is obtained from X using $y = (x-2)^2$.What are the CDF and PDF for Y?</td><td>L2, L3</td><td>6</td></tr>
<tr><td colspan="3">Module-2</td><td></td><td></td></tr>
<tr><td>Q. 03</td><td>a</td><td>The joint PDF $f_{XY}(x,y) = c$, a constant, when $(0 < x < 3)$ and $(0 < y < 4)$ and is 0 otherwise.
(i) What is the value of the constant c?
(ii) What are the PDFs for X and Y?
(iii) What is $F_{XY}(x,y)$when $(0 < x < 3)$ and $(0 < y < 4)$?
(iv) What are $F_{XY}(x,\infty)$ and $F_{XY}(\infty,y)$?
(v) Are X and Y independent?</td><td>L1, L2, L3</td><td>10</td></tr>
<tr><td></td><td>b</td><td>Define correlation coefficient of random variables X and Y. Show that it is bounded by limits ± 1.</td><td>L1, L2</td><td>5</td></tr>
</table>

	c	X is a random variable with $\mu_X = 4$ and $\sigma_X = 5$. Y is a random variable with $\mu_Y = 6$, and $\sigma_Y = 7$. The correlation coefficient is 0.7. If U = 3X + 2Y, what are Var[U], Cov [UX] and Cov [UY]?	L3	5
OR				
Q.04	a	X is a random variable uniformly distributed between 0 and 3. Y is a random variable, independent of X, uniformly distributed between $+2$ and -2. W = X + Y. What is the PDF for W?	L2, L3	8
	b	The random variable Z is uniformly distributed between 0 and 1. The random variable Y is obtained from Z as follows: Y = 3.5Z + 5.25 One hundred independent realizations of Y are averaged: $$V = \frac{1}{100}\sum_{i=1}^{100} Y_i$$ (i) Estimate the probability $P(V \leq 7.1)$ (ii) If 1000 independent calculations of V are performed, approximately how many of these calculated values for V would be less than7.1?	L3, L4	8
	c	Explain briefly the following random variables. (i) Chi-Square Random Variable (ii) Student's t Random Variable	L1	4
Module-3				
Q. 05	a	With the help of an example, define Random Process and discuss distributions and density functions of a random process.	L1	5
	b	A random process is described by $$X(t) = A\cos(\omega_c t + \varphi + \theta)$$ Where A, ω_c and φ are constants and where θ is a random variable uniformly distributed between $\pm\pi$. Is $X(t)$ wide-sense stationary? If not, then why not? If so, then what are the mean and the autocorrelation function for the random process?	L2, L3	8
	c	Define the autocorrelation function (ACF) of a random process and discuss its properties.	L1, L2	7
OR				
Q. 06	a	$X(t)$ and $Y(t)$ are independent, jointly wide-sense stationary random processes given by, $X(t) = A\cos(\omega_1 t + \theta_1)$ and $Y(t) = B\cos(\omega_2 t + \theta_2)$. If $W(t) = X(t)Y(t)$ then find the ACF $R_W(\tau)$.	L3	6
	b	Assume that the data in the following table are obtained from a windowed sample function obtained from an ergodic random process. Estimate the ACF for $\tau = 0, 2\ ms$ and $4\ ms$, where $\Delta t = 2\ ms$. $x(t)$: 1.5, 2.1, 1.0, 2.2, -1.6, -2.0, -2.5, 2.5, 1.6, -1.8 k: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9	L2, L3	6
	c	Suppose that the PSD input to a linear system is $S_X(\omega) = K$. The cross-correlation of the input X(t) with the output Y(t) of the linear system is found to be $$R_{XY}(\tau) = K\begin{cases} e^{-\tau} + 3e^{-2\tau}; & \tau \geq 0 \\ 0; & \tau < 0 \end{cases}$$ What is the power filter function $\lvert H(j\omega)\rvert^2$?	L3, L4	8
Module-4				
Q. 07	a	Describe the column space and the null space of the following matrices. (i) $A = \begin{bmatrix} 1 & -1 \\ 0 & 0 \end{bmatrix}$ (ii) $B = \begin{bmatrix} 0 & 0 & 3 \\ 1 & 2 & 3 \end{bmatrix}$	L2, L3	4
	b	Determine whether the vectors (1, 3, 2), (2, 1, 3) and (3, 2, 1) are linearly dependent or independent.	L3	6

	c	If $u = \begin{bmatrix}1\\2\\2\end{bmatrix}$, $v = \begin{bmatrix}2\\-2\\1\end{bmatrix}$ and $w = \begin{bmatrix}2\\1\\-2\end{bmatrix}$ then show that *u*, *v*, *w* are pairwise orthogonal vectors. Find lengths of *u*, *v*, *w* and find orthonormal vectors u_1, v_1, w_1 from vectors *u*, *v*, *w*.	L2, L3	10
OR				
Q. 08	a	Apply Gram-Schmidt process to $a = \begin{bmatrix}0\\0\\1\end{bmatrix}$, $b = \begin{bmatrix}0\\1\\1\end{bmatrix}$ and $c = \begin{bmatrix}1\\1\\1\end{bmatrix}$ and write the result in the form of A = QR.	L3	8
	b	Find the dimension and basis for four fundamental subspaces for $$A = \begin{bmatrix}1 & 2 & 0 & 1\\0 & 1 & 1 & 0\\1 & 2 & 0 & 1\end{bmatrix}$$	L3	8
	c	Find the projection of *b* onto the column space of A. $A = \begin{bmatrix}1 & 1\\1 & -1\\-2 & 4\end{bmatrix}$ and $b = \begin{bmatrix}1\\2\\7\end{bmatrix}$	L3	4
Module-5				
Q. 09	a	(i) Reduce the matrix A to U and find *det (A)* using pivots of A. $$A = \begin{bmatrix}1 & 2 & 3\\2 & 2 & 3\\3 & 3 & 3\end{bmatrix}$$ (ii) By applying row operations to produce an upper triangular matrix U, compute the *det (A)*. $$A = \begin{bmatrix}1 & 2 & 3 & 0\\2 & 6 & 6 & 1\\-1 & 0 & 0 & 3\\0 & 2 & 0 & 7\end{bmatrix}$$	L3	6
	b	Find the eigen values and eigen vectors of matrix A. $$A = \begin{bmatrix}1 & 4\\2 & 3\end{bmatrix}$$	L3	6
	c	Factor the matrix A into $A = X\Lambda X^{-1}$ using diagonalization and hence find A^3. $$A = \begin{bmatrix}1 & 2\\0 & 3\end{bmatrix}$$	L3	8
OR				
Q. 10	a	Factorize the matrix A into $A = U\Sigma V^T$ using SVD. $$A = \begin{bmatrix}1 & 1 & 0\\0 & 1 & 1\end{bmatrix}$$	L3, L4	8
	b	(i) What is a positive definite matrix? Mention the methods of testing positive definiteness. (ii) Check the following matrix for positive definiteness. $$S_1 = \begin{pmatrix}5 & 6\\6 & 7\end{pmatrix}$$	L1, L2	6
	c	Find an orthogonal matrix Q that diagonalizes the following symmetric matrix. $$S = \begin{bmatrix}1 & 0 & 2\\0 & -1 & -2\\2 & -2 & 0\end{bmatrix}$$	L3	6