Model Question Paper-2 with effect from 2019-20 (CBCS Scheme)

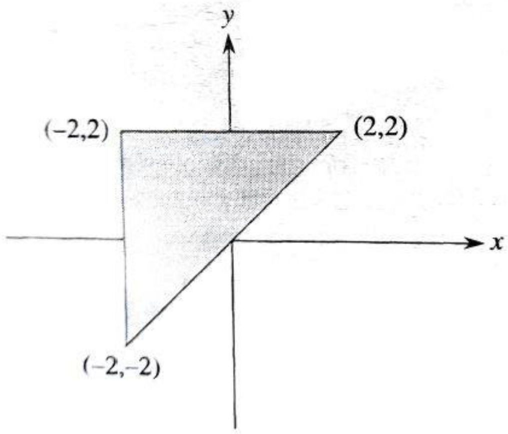
USN | | | | | | | | | |

Fourth Semester B.E. Degree Examination
Engineering Statistics & Linear Algebra

TIME: 03 Hours **Max. Marks: 100**

Note: 01. Answer any **FIVE** full questions, choosing at least **ONE** question from each **MODULE**.
02. Use of Normalized Gaussian Random Variables table is permitted.

		Module -1	***Bloom's Taxonomy Level**	**Marks**
Q.01	a	Define a random variable and briefly discuss the following terms associated with random variables. (i) Sample space (ii) Distribution Function (iii) Probability Mass Function (iv) Probability Density Function	L1	5
	b	The probability distribution of a Discrete Random Variable (DRV) is as shown below. k: -0.25, 0, 1, 2, 3.75 $P(X=k)$: 0.2, c, 0.4, 0.1, 2c Find (i) the value of c. (ii) $P(x \leq 0)$ (iii) $P\{(X > 1)/(X \geq 0)\}$	L2, L3	7
	c	Define Binomial distribution. Obtain the characteristic function of a binomial random variable and using the characteristic function derive its mean and variance.	L1, L3	8
		OR		
Q.02	a	The random variable X is uniformly distributed between 0 and 2. If $y = 3x^3$, then find the PDF for Y.	L3	5
	b	Let X be an exponential random variable with CDF $F_X(x) = \begin{cases} 1-\exp\left(-\frac{x}{3}\right), & x \geq 0 \\ 0, & x < 0 \end{cases}$ And let B be the event $B = \{x > 2\}$. What are $f_{\{X\|B\}}(x)$, $\mu_{\{X\|B\}}(x)$ and $\sigma^2_{(X\|B)}$?	L3, L4	7
	c	Define Laplace distribution. Obtain the characteristic function of a Laplace random variable and using the characteristic function derive its mean and variance.	L1, L2, L3	8
		Module-2		
Q. 03	a	A bivariate PDF for the DRVs X and Y is $0.2\,\delta(x)\delta(y) + 0.3\,\delta(x-1)\delta(y) + 0.3\,\delta(x)\delta(y-1) + c\,\delta(x-1)\delta(y-1)$. (i) What is the value of the constant c? (ii) What are the PDFs for X and Y? (iii) What is $F_{XY}(x, y)$ when $(0 < x < 1)$ and $(0 < y < 1)$? (iv) What are marginal CDFs of X and Y? (v) Are X and Y independent?	L2, L3	5
	b	X and Y are correlated random variables with a correlation coefficient of 0.7, mean of X is 5, variance of X is 36, mean of Y is 16, variance of Y is 150. The random variables U and V are obtained using	L2, L3, L4	7

		$U = X + cY$ and $V = X - cY$ What values can c have if U and V are uncorrelated?		
	c	Briefly explain the following random variables (RV). (i) Chi-Square RV (ii) Student's t RV (iii) Cauchy RV (iv) Rayleigh RV	L1	8
OR				
Q.04	a	A DRV Y has the PDF $f_Y(y) = 0.5\,\delta(y) + 0.5\,\delta(y-3)$ $U = Y_1 + Y_2$,where the Y's are independent. What is the PDF for U?	L1, L2	5
	b	Shown in Fig. Q4c is a region in the x, y plane where the bivariate pdf $f_{XY}(x,y) = c$, elsewhere, the pdf is 0.  Fig. Q4c (i) What value must c have? (ii) Evaluate $F_{XY}(1,1)$. (iii) Find the pdfs of X and Y.	L4, L5	7
	c	Define Central Limit Theorem and show that the sum of the two independent Gaussian random variables is also Gaussian.	L1, L3	8
Module-3				
Q. 05	a	With the help of an example, define Random Process and discuss the terms Strict-Sense Stationary (SSS) and Wide-Sense Stationary (WSS) associated with a random process.	L1	5
	b	A random process is described by $X(t) = A\cos(\omega_c t + \theta) + B$ Where A, B, ω_c are constants and where θ is a random variable uniformly distributed between $\pm\pi$. Is $X(t)$ wide-sense stationary? If not, then why not? If so, then what are the mean and the autocorrelation function for the random process?	L2, L3	7
	c	Define the Autocorrelation function (ACF) of the random process X(t). And prove the following statements. (i) ACF is an even function. (ii) If X(t) is periodic with period T, then in the WSS case, the ACF is also periodic with period T.	L1, L2	8
OR				
Q. 06	a	$X(t)$ and $Y(t)$ are independent, jointly wide-sense stationary random processes given by, $X(t) = A\cos(\omega_1 t + \theta_1)$ and $Y(t) = B\cos(\omega_2 t + \theta_2)$. If $W(t) = X(t)Y(t)$ then find the ACF $R_W(\tau)$.	L3	5
	b	Assume that the data in the following table are obtained from a windowed sample function obtained from an ergodic random process. Estimate the ACF for $\tau = 0, 3\,ms$ and $6\,ms$, where $\Delta t = 3\,ms$.	L2, L3	7

		$x(t)$: 1.0, 2.2, 1.5, −3.0, −0.5, 1.7, −3.5, −1.5, 1.6, −1.3; k: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9		
	c	Suppose that the PSD input to a linear system is $S_X(\omega) = K$. The cross-correlation of the input X(t) with the output Y(t) of the linear system is found to be $$R_{XY}(\tau) = K\begin{cases} 3e^{-\tau} + e^{-2\tau}; & \tau \ge 0 \\ 0; & \tau < 0 \end{cases}$$ What is the power filter function $\lvert H(j\omega)\rvert^2$?	L3, L4	8
		Module-4		
Q. 07	a	Let $I: V \to \mathbb{R}$ be the integral mapping $I(v) = \int_0^1 v(t)\,dt$. Show that I is a linear transformation.	L2, L3	5
	b	Determine whether or not each of the following forms a basis in $\mathbb{R}^3$. $x_1 = (2, 2, 1), x_2 = (1, 3, 7)$ and $x_3 = (1, 2, 2)$.	L3, L4	7
	c	If $u = \begin{bmatrix}1\\2\\2\end{bmatrix}, v = \begin{bmatrix}2\\-2\\1\end{bmatrix}$ and $w = \begin{bmatrix}2\\1\\-2\end{bmatrix}$ then show that *u, v, w* are pairwise orthogonal vectors. Find lengths of *u, v, w* and find orthonormal vectors u_1, v_1, w_1 from vectors *u, v, w.*	L2, L3	8
		OR		
Q. 08	a	Define Vector Subspaces and explain the four fundamental subspaces.	L1	5
	b	Solve $Ax = b$ by least squares and find $p = A\hat{x}$, if $A = \begin{bmatrix}1 & 0\\0 & 1\\1 & 1\end{bmatrix}, b = \begin{bmatrix}1\\1\\0\end{bmatrix}$	L3	7
	c	Apply Gram-Schmidt process to $a = \begin{bmatrix}2\\2\\1\end{bmatrix}, b = \begin{bmatrix}1\\3\\1\end{bmatrix}$ and $c = \begin{bmatrix}1\\2\\2\end{bmatrix}$ and write the result in the form of A = QR.	L3	8
		Module-5		
Q. 09	a	(i) Reduce the matrix A to U and find *det (A)* using pivots of A. $$A = \begin{bmatrix}2 & 5 & 3\\1 & 2 & 4\\-1 & 3 & 6\end{bmatrix}$$ (ii) By applying row operations to produce an upper triangular matrix U, compute the *det (A).* $$A = \begin{bmatrix}3 & 1 & 4 & 2\\1 & 5 & 2 & 6\\2 & 3 & 7 & 1\\4 & 1 & 2 & 3\end{bmatrix}$$	L3	5
	b	Find the eigen values and eigen vectors of matrix A and A^{-1} $A = \begin{bmatrix}0 & 2\\1 & 1\end{bmatrix}$ and $A^{-1} = \begin{bmatrix}-\frac{1}{2} & 1\\ \frac{1}{2} & 0\end{bmatrix}$ Check the trace. Comment on the results.	L3	7
	c	Diagonalize the matrix A and hence find A^4. $$A = \begin{bmatrix}1 & 6 & 1\\1 & 2 & 0\\0 & 0 & 3\end{bmatrix}$$	L3	8
		OR		

Q. 10	a	If a 4×4 matrix has $\det(A) = \frac{1}{2}$ then find the following. (i) $det\ (2A)$ (ii) $\det(-A)$ (iii) $\det(A^2)$ (iv) $\det\ (A^{-1})$	L2, L3	5
	b	Test to see if $A^T A$ is positive definite $A = \begin{bmatrix} 1 & 1 \\ 1 & 2 \\ 2 & 1 \end{bmatrix}$	L4	7
	c	Compute $A^T A$ and AA^T and their eigen values & unit eigen vectors for V and U. Then check $AV = U\Sigma$. $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \\ -1 & 1 \end{bmatrix}$	L4	8