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First/Second Semester B.E. Degree(CBCS)Examination

Engineering Mathematics-I

Time: 3 hrs.

Max. Marks: 80

Note: Answer any FIVE full questions, choosing one full question from each module.

**Module-1**

- 1 a. Find the nth derivative of  $\frac{x^2+4x+1}{x^3+2x^2-x-2}$  (04 Marks)
- b. Find the angle between the radius vector and the tangent for the curve  $r = a(1 + \cos\theta)$  and also find the slope of the tangent at  $\theta = \frac{\pi}{3}$  (04 Marks)
- c. Find the angle of intersection between the curves  $r^2 \sin 2\theta = 4$  and  $r^2 = 16 \sin 2\theta$ . (04 Marks)
- d. Obtain the Pedal equation of the curve  $\frac{2a}{r} = 1 + \cos\theta$  (04 Marks)

OR

- 2 a. If  $y = e^{m \sin^{-1} x}$ , prove that  $(1 - x^2)y_{n+2} - (2n + 1)xy_{n+1} - (m^2 + n^2)y_n = 0$  (06 Marks)
- b. Find the pedal equation,  $r^n = a^n \cos n\theta$  (05 Marks)
- c. Show that the radius of curvature of the curve  $x^3 + y^3 = 3axy$  at  $(\frac{3a}{2}, \frac{3a}{2})$  is  $-\frac{3a}{8\sqrt{2}}$  (05 Marks)

**Module-2**

- 3 a. Expand  $\log_e x$  in powers of  $(x - 1)$  up to fourth degree term. (04 Marks)
- b. Evaluate  $\lim_{x \rightarrow 0} \left(\frac{a^x + b^x + c^x}{3}\right)^{1/x}$  (04 Marks)
- c. If  $z = f(x + ct) + g(x - ct)$  prove that  $\frac{\partial^2 z}{\partial t^2} = c^2 \frac{\partial^2 z}{\partial x^2}$  (04 Marks)
- d. If  $u = \cos^{-1} \left(\frac{x^2 + y^2}{x + y}\right)$ , then prove that  $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = -2cotu$  (04 Marks)

OR

- 4 a. Prove that  $\sqrt{1 + \sin 2x} = 1 + x - \frac{x^2}{2} - \frac{x^3}{3} + \frac{x^4}{24} + \dots$  (06 Marks)
- b. If  $u = f(x-y, y-z, z-x)$  then prove that  $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$  (05 Marks)
- c. If  $u = x + y + z, v = x^2 + y^2 + z^2, w = xy + yz + zx$ , then find  $\frac{\partial(u,v,w)}{\partial(x,y,z)}$  (05 Marks)

**Module-3**

- 5 a. A particle moves along the curve  $x=1-t^3, y=1+t^2$  and  $z= 2t-5$ , find the velocity and acceleration. Also find the components of velocity and acceleration at  $t=1$  in the direction  $2i+j+2k$ . (08 Marks)
  - b. Find the constants  $a, b, c$  so that the vector field  $\vec{F} = (x + 2y + az)\hat{i} + (bx - 3y - z)\hat{j} + (4x + cy + 2z)\hat{k}$  is irrotational. (08 Marks)
- OR
- 6 a. If  $\vec{u} = x^2\hat{i} + y^2\hat{j} + z^2\hat{k}$  and  $\vec{v} = yz\hat{i} + xz\hat{j} + xy\hat{k}$  then prove that  $\vec{u} \times \vec{v}$  is a solenoidal vector. (08 Marks)
  - b. Find  $div \vec{F}$  and  $curl \vec{F}$  if  $\vec{F} = grad(x^3 + y^3 + z^3 - 3xyz)$ . (08 Marks)

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.  
2. Any revealing of identification, appeal to evaluator and /or equations written eg. 42+8 = 50, will be treated as malpractice.

**Module-4**

- 7 a. Obtain the reduction formula for  $\int_0^{\pi/2} \sin^n x \, dx$ . (06 Marks)  
 b. Solve  $(y^2 e^{xy^2} + 4x^3)dx + (2xye^{xy^2} - 3y^2)dy = 0$ . (05 Marks)  
 c. Find the Orthogonal trajectories of the family of cardioids  $r = a(1 + \cos\theta)$  where  $a$  is the parameter. (05 Marks)

OR

- 8 a. Evaluate  $\int_0^{\pi/6} \cos^4 3\theta \sin^3 6\theta \, d\theta$ . (06 Marks)  
 b. Solve  $x \frac{dy}{dx} + y = x^3 y^6$  (05 Marks)  
 c. If a substance cools from 370k to 330k in 10minutes, when the temperature of the surrounding air is 290k. Find the temperature of the substance after 40 minutes. (05 Marks)

**Module-5**

- 9 a. Solve the following system of equations by Gauss elimination method

$$x + 2y + z = 3$$

$$2x + 3y + 3z = 10$$

$$3x - y + 3z = 13$$

(06 Marks)

- b. Use power method to find the largest eigen value and the corresponding eigen vector of the matrix taking  $[0 \ 1]^T$  as initial eigen vector

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

(05 Marks)

- c. Show that the transformation  $y_1 = x - y + z$

$$y_2 = 3x - y + 2z$$

$$y_3 = 2x - 2y + 3z$$

is non-singular. Find the inverse transformation

(05 Marks)

OR

- 10 a. Solve the following system of equations by Gauss-Seidel method

$$20x + y - 2z = 17$$

$$3x + 20y - z = -18$$

$$2x - 3y + 20z = 25$$

(05 Marks)

- b. Reduce the following matrix to the diagonal form

$$\begin{bmatrix} -19 & 7 \\ -42 & 16 \end{bmatrix}$$

(05 Marks)

- c. Reduce the quadratic form  $2xy + 2xz - 2yz$  to the canonical form by orthogonal transformation. (06 Marks)