Model Question Paper-1 with effect from 2019-20 (CBCS Scheme)

USN | | | | | | | | | | | **18MAT31**

Third Semester B.E.Degree Examination
Transform Calculus, Fourier Series and Numerical Techniques

(Common to all Programmes)

Time: 3 Hrs   Max.Marks: 100

Note: Answer any FIVE full questions, choosing at least ONE question from each module.

Module-1

1. (a) Find the Laplace transform of (i) $\sqrt{e^{4(t+3)}}+e^{-2t}\sin 3t$ (ii) $te^{-3t}\sin 4t$ (iii) $(1-\cos t)/t$ **(10 Marks)**

   (b) The square wave function $f(t)$ with period "a" is defined by $f(t)=\begin{cases} E, & 0\le t<a/2 \\ -E, & a/2\le t<a. \end{cases}$

   Show that $L\{f(t)\}=(E/s)\tanh(as/4)$. **(05 Marks)**

   (c) Employ Laplace transform to solve $\frac{d^2y}{dx^2}-3\frac{dy}{dx}-4y=2e^{-x}, y(0)=1=y'(0)$. **(05 Marks)**

OR

2. (a) Find (i) $L^{-1}\left\{\frac{3s+2}{s^2-s-2}\right\}$ (ii) $L^{-1}\{(s+5)/(s^2-6s+13)\}$ (iii) $L^{-1}[\cot^{-1}\{s/a\}]$ **(10 Marks)**

   (b) Express $f(t)=\begin{cases} 1, & 0\le t\le 1 \\ t, & t>1 \end{cases}$ in terms Heaviside's unit step function and hence find its Laplace transform. **(05 Marks)**

   (c) Find the inverse Laplace transform of $\frac{s^2}{(s^2+a^2)(s^2+b^2)}$, using convolution theorem. **(05 Marks)**

Module-2

3. (a) Find the Fourier series expansion of $f(x)=\frac{\pi^2}{12}-\frac{x^2}{4}$ in $-\pi\le x\le\pi$. Hence deduce that

   $\frac{1}{1^2}-\frac{1}{2^2}+\frac{1}{3^2}-\frac{1}{4^2}+...=\frac{\pi^2}{12}$. **(07 Marks)**

   (b) Find the half-range cosine series of $f(x)=(x+1)^2$ the interval $0\le x\le 1$. **(06 Marks)**

   (c) Obtain the Fourier series of $f(x)=\begin{cases} l-x, & for\ 0\le x\le l \\ 0, & for\ l\le x\le 2l \end{cases}$ Hence deduce that

   $\frac{1}{1}-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+...=\frac{\pi}{4}$. **(07 Marks)**

OR

4. (a) The displacement y (in cms) of a machine part occurs due to the rotation of x radians is given below:

Rotation x (in radians)	0	$\pi/3$	$2\pi/3$	π	$4\pi/3$	$5\pi/3$	2π
Displacement y (in cms)	1.0	1.4	1.9	1.7	1.5	1.2	1.0

Expand y in terms of Fourier series up to second harmonics. **(07 Marks)**

(b) Find the half-range sine series of e^x the interval $0 \le x \le 1$. **(06 Marks)**

(c) Find the Fourier series expansion of $f(x) = |x|$ in $-\pi \le x \le \pi$. Hence deduce that

$$\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} + \ldots = \frac{\pi^2}{8}.$$ **(07 Marks)**

Module-3

5. (a) If $f(x) = \begin{cases} 1 - x^2, & for \quad |x| \le 1 \\ 0 & for \quad |x| > 1 \end{cases}$, find the infinite Fourier transform of *f(x)* and hence evaluate

$$\int_0^\infty \frac{x\cos x - \sin x}{x^3} \cos\frac{x}{2}\, dx$$ **(07 Marks)**

(b) Find the Fourier cosine transform of $f(x) = e^{-2x} + 4e^{-3x}$ **(06 Marks)**

(c) Solve: $u_{n+2} - 3u_{n+1} + 2u_n = 2^n$, given $u_0 = 0, u_1 = 1$ by using z-transforms. **(07 Marks)**

OR

6. (a) Find the Fourier sine transform of $e^{-|x|}$. Hence show that $\int_0^\infty \frac{x\sin mx}{1+x^2} dx = \frac{\pi e^{-m}}{2}, m. > 0.$ **(07 Marks)**

(b) Find the z-transform of $\cos[n\pi/2 + \pi/4]$ **(06 Marks)**

(c) Find the inverse z-transform of $\frac{2z^2 + 3z}{(z+2)(z-4)}$ **(07 Marks)**

Module-4

7. (a) Solve $\frac{dy}{dx} = e^x - y, y(0) = 1$ using Taylor's series method considering up to fourth degree terms and, find the value of $y(0.1)$. **(07 Marks)**

(b) Use Runge - Kutta method of fourth order to solve $(x + y)\frac{dy}{dx} = 1, y(0.4) = 1$, to find $y(0.5)$. (Take $h = 0.1$). **(06 Marks)**

(c) Given that $\frac{dy}{dx}+\frac{y}{x}=\frac{1}{x^2}$ and $y(1)=1, y(1.1)=0.9960, y(1.2)=0.9860, \& \ y(1.3)=0.9720$ find $y(1.4)$, using Adam-Bashforth predictor-corrector method. **(07 Marks)**

OR

8. (a) Solve the differential equation $\frac{dy}{dx}=x\sqrt{y}$ under the initial condition $y(1)=1$, by using modified Euler's method at the point $x=1.4$. Perform three iterations at each step, taking $h=0.2$. **(07 Marks)**

(b) Use fourth order Runge - Kutta method, to find $y(0.1)$ with $h=0.1$, given

$\frac{dy}{dx}+y+xy^2=0, y(0)=1$, **(06 Marks)**

(c) Apply Milne's predictor-corrector formulae to compute $y(0.3)$ given, $\frac{dy}{dx}=x+y^2$ with **(07 Marks)**

x	0.0	0.1	0.2	0.3
y	1.0000	1.1000	1.2310	1.4020

Module-5

9. (a) Solve $\frac{d^2y}{dx^2}-x^2\frac{dy}{dx}-2xy=1$, for $x=0.1$, correct to four decimal places, using initial conditions $y(0)=1, y'(0)=0$, using Runge - Kutta method , **(07 Marks)**

(b) Find the extremal of the functional $\int_0^1 \left(y'^2-y^2-y\right)e^{2x}dx$, that passes through the points $(0,0)$ and $(1,1/e)$. **(06 Marks)**

(c) A heavy cable hangs freely under gravity at two fixed points. Show that the shape of the cable is catenary. **(07 Marks)**

OR

10. (a) Apply Milne's predictor-corrector method to compute $y(0.4)$ given the differential equation $\frac{d^2y}{dx^2}=1+\frac{dy}{dx}$ and the following table of initial values: **(07 Marks)**

x	0	0.1	0.2	0.3
y	1	1.1103	1.2427	1.3990
y'	1	1.2103	1.4427	1.6990

(b) Derive Euler's equation in the standard form viz., $\frac{\partial f}{\partial y}-\frac{d}{dx}\left[\frac{\partial f}{\partial y'}\right]=0$ **(06 Marks)**

(c) Find the extremal for the functional $\int_0^{\pi/2}\left(y^2-y'^2-2y\sin x\right)dx$; $y(0)=0, y(\pi/2)=1$. **(07 Marks)**

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