Model Question Paper-I with effect from 2022

USN | | | | | | | | | | |

Third Semester B.E Degree Examination

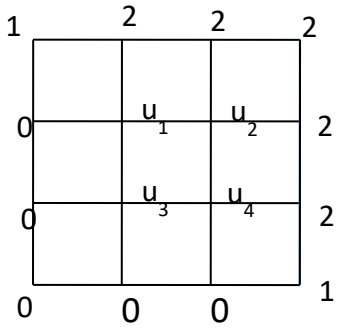
Transform Calculus, Fourier Series and Numerical Techniques (21MAT31)

TIME: 03 Hours **Max. Marks: 100**

Note: Answer any **FIVE** full questions, choosing at least **ONE** question from each module.

		Module -1	**Marks**
Q.01	a	Find the Laplace transform of $te^{-t}sin2t + \frac{cos2t-cos3t}{t}$	**06**
	b	Find the Laplace transform of the triangular wave of period 2a given by $f(t) = \begin{cases} t, & 0 < t < a \\ 2a - t, & a < t < 2a \end{cases}$	**07**
	b	Using convolution theorem find the inverse Laplace transform of $\frac{s}{(s^2+a^2)^2}$	**07**
		OR	
Q.02	a	Find the inverse Laplace transform of (i) $\frac{(s^2-1)^2}{s^5}$ (ii) $\frac{s}{s^2+6s+13}$	**06**
	b	Express the following function in terms of unit step function and hence find its Laplace transform $f(t) = \begin{cases} 1, & 0 < t < 1 \\ 2t, & 1 < t < 2 \\ 3t, & 2 < t < 3 \end{cases}$	**07**
	c	Solve by using Laplace transform techniques $y'' - 3y' + 2y = e^{3t}, y(0) = 1, \ y'(0) = -1$	**07**
		Module-2	
Q. 03	a	Obtain the Fourier series for $f(x) = \frac{\pi - x}{2} \ in \ 0 \le x \le 2\pi$	**06**
	b	Find half-range Fourier cosine series for the function $f(x) = (x-1)^2, in \ 0 < x < 1$, and hence show that $\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots$	**07**
	c	Find Fourier series expansion of y up to first harmonic if it is given by	**07**
		OR	
Q.04	a	Obtain the Fourier series for $f(x) = \lvert x \rvert, \ -\pi \le x \le \pi$	**6**
	b	Obtain half-range sine series for $f(x) = \begin{cases} x, & 0 \le x \le \frac{\pi}{2} \\ \pi - x, & \frac{\pi}{2} \le x \le \pi \end{cases}$	**07**

x	0	1	2	3	4	5
f(*x*)	9	18	24	28	26	20

	c	Expand y as a Fourier series up to first harmonic if the values of y given by x: 0 | $\pi/6$ | $\pi/3$ | $\pi/2$ | $2\pi/3$ | $5\pi/6$ y: 1.98 | 1.30 | 1.05 | 1.30 | -0.88 | -0.25	**07**
		Module-3	
Q. 05	a	Find the Fourier transform of $f(x) = \begin{cases} 1-x^2, & \lvert x\rvert \le 1 \\ 0, & \lvert x\rvert > 1 \end{cases}$ Hence evaluate $\int_0^\infty \frac{sinx - x\,cosx}{x^3}\cos\left(\frac{x}{2}\right)dx$	**06**
	b	Find the Z-transforms of $\cosh n\theta$ and $\sinh n\theta$	**07**
	c	Using z –transformation, solve the difference equation $u_{n+2} + 6u_{n+1} + 9u_n = 2^n$, $u_0 = 0, u_1 = 0$	**07**
		OR	
Q. 06	a	Find the Fourier sine transform of $f(x) = \begin{cases} x, & 0 < x < 1 \\ 2-x, & 1 < x < 2 \\ 0, & x > 2 \end{cases}$	**06**
	b	Fin the inverse cosine transform of $F_c(\alpha) = \begin{cases} 1-\alpha, & 0 \le \alpha \le 1 \\ 0, & \alpha > 1 \end{cases}$ And hence evaluate $\int_0^\infty \left(\frac{\sin t}{t}\right)^2 dt$	**07**
	c	Fin the inverse z-transform of $\frac{z^2-20z}{(Z-2)(Z-3)(z-4)}$	**07**
		Module-4	
Q. 07	a	Solve $u_{xx} + u_{yy} = 0$ for the square mesh with boundary values as given below. Iterate till the mesh values are correct to two decimal places 	**10**
	b	Evaluate the pivotal values of the equation $u_{tt} = 16u_{xx}$, taking h = 1 up to t = 1.25. Ihe boundary condition are $u(0,t) = u(5,t) = 0, u_t(x,0) = 0$ and $u(x,0) = x^2(5-x)$	**10**
		OR	

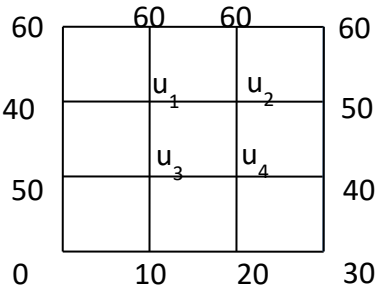
Q. 08	a	Given the values of u(x, y) on the boundary of the square as in the following figure. Evaluate the function u(x, y) satisfying the Laplace equation $u_{xx} + u_{yy} = 0$ at the pivotal points of the figure 	**10**
	b	Find the solution of the parabolic equation $u_{xx} = 2u_t$ when $u(4, t) = 0, and\ u(x, 0) = x(4 - x)$, taking $h = 1$. Find the values up to t = 5.	**10**
		Module-5	
Q. 09	a	Using Runge –Kutta method of order four, solve $\frac{d^2y}{dx^2} = x\left(\frac{dy}{dx}\right)^2 - y^2$ for x = 0.2 . Given, $y(0) = 1,\ y'(0) = 1$	**06**
	b	Find the external of the functional $\int_{x_0}^{x_1}(1 + x^2y')y'dx$	**07**
	c	Show that the geodesies on a plane are straight lines	**07**
		OR	
Q. 10	a	Given $y'' = 1 + y', y(0) = 1, y'(0) = 1$, compute y(0.4) for the following data using Milne's predictor – corrector method. $y(0.1) = 1.1103,\quad y(0.2).2427,\quad y(0.3) = 1.344$ $y'(0.1) = 1.2103,\ y'(0.2) = 1.4427,\ y'(0.3) = 1.699$	06
	b	Derive the Euler's equation	**07**
	c	Find the curves on which the functional $\int_0^1[(y')^2 + 12xy]\,dx$ with $y(0) = 0\ and\ y(1) = 1$	**07**

Table showing the Bloom's Taxonomy Level, Course Outcome and Program Outcome

Question		Bloom's Taxonomy Level attached	Course Outcome	Program Outcome
Q.1	(a)	L1	CO 01	PO01
	(b)	L2	CO 01	PO 02
	(c)	L3	CO 01	PO 01
Q.2	(a)	L1	CO 01	PO 01
	(b)	L2	CO 01	PO 02
	(c)	L3	CO 01	PO 03

Q.3	(a)	L2	CO 02	PO 01
	(b)	L2	CO 02	PO 01
	(c)	L3	CO 02	PO 02
Q.4	(a)	L2	CO 02	PO 01
	(b)	L2	CO 02	PO 01
	(c)	L3	CO 02	PO 02
Q.5	(a)	L2	CO 03	PO 02
	(b)	L2	CO 03	PO 02
	(c)	L3	CO 03	PO 03
Q.6	(a)	L2	CO 03	PO 02
	(b)	L2	CO 03	PO 02
	(c)	L3	CO 03	PO 02
Q.7	(a)	L3	CO 04	PO 03
	(b)	L3	CO 04	PO 03
Q.8	(a)	L3	CO 04	PO 03
	(b)	L3	CO 04	PO 03
Q.9	(a)	L2	CO 05	PO 02
	(b)	L2	CO 05	PO 02
	(c)	L2	CO 05	PO 02
Q.10	(a)	L2	CO 05	PO 02
	(b)	L2	CO 05	PO 02
	(c)	L2	CO 05	PO 02

Bloom's Taxonomy Levels	**Lower order thinking skills**		
	Remembering (knowledge): L_1	Understanding (Comprehension): L_2	Applying (Application): L_3
	Higher order thinking skills		
	Analyzing (Analysis):L_4	Valuating (Evaluation): L_5	Creating (Synthesis): L_6