Model Question Paper-I with effect from 2021 (CBCS Scheme)

USN | | | | | | | | | | |

First Semester B.E Degree Examination

Calculus and Differential Equations (21MAT11)

TIME: 03 Hours **Max. Marks: 100**

Note: Answer any **FIVE** full questions, choosing at least **ONE** question from each **MODULE**.

		Module -1	**Marks**
Q.01	a	With usual notations prove that $\tan\varphi = r\frac{d\theta}{dr}$	**06**
	b	Find the angle between the curves $r = a(1+\cos\theta)$ *and* $r = b(1-\cos\theta)$	**07**
	c	Show that the radius of curvature at any point of the cycloid $x = a(\theta = \sin\theta),\ y = a(1-\cos\theta)$ *is* $4a\cos\left(\frac{\theta}{2}\right)$	**07**
		OR	
Q.02	a	If p be the perpendicular from the pole on the tangent, then show that $\frac{1}{p^2} = \frac{1}{r^2} + \frac{1}{r^4}\left(\frac{dr}{d\theta}\right)^2$	**06**
	b	Find the pedal equation of the curve $r^m = a^m(\cos m\theta + \sin m\theta)$	**07**
	c	Find the radius of curvature of the curve $x^3 + y^3 = 3axy$ at $\left(\frac{3a}{2}, \frac{3a}{2}\right)$	**07**
		Module-2	
Q. 03	a	Expand e^{sinx} by Maclaurin's series up to the term containing x^4	**06**
	b	If $u = f(2x-3y, 3y-4z, 4z-2x)$, show that $6u_x + 4u_y + 3u_z = 0$	**07**
	c	Examine the function $f(x,y) = xy(1-x-y)$ for extreme values	**07**
		OR	
Q.04	a	Evaluate (i) $\lim_{x\to 0}\left(\frac{tanx}{x}\right)^{\frac{1}{x^2}}$ (ii) $\lim_{x\to 0}(\cos x)^{\frac{1}{x^2}}$	**06**
	b	If $z = e^{ax+by}f(ax-by)$, show that $b\frac{\partial z}{\partial x} + a\frac{\partial z}{\partial y} = 2abz$	**07**
	c	If $x+y+z = u,\ y+z = uv$ *and* $z = uvw,$ *find* $\frac{\partial(x,y,z)}{\partial(u,v,w)}$	**07**

Module-3			
Q. 05	a	Solve $x\frac{dy}{dx} + y = x^3y^6$	**06**
	b	Find the orthogonal trajectories of the family of curves $\frac{x^2}{a^2} + \frac{y^2}{b^2+\lambda} = 1$, where λ *is a parameter*	**07**
	c	Solve $xyp^2 - (x^2 + y^2)p + xy = 0$	**07**
OR			
Q. 06	a	Solve $(x^2 + y^2 + x)dx + xydy = 0$	**06**
	b	A copper ball originally at $80°$ C cools down to $60°C$ in 20 minutes, if the temperature of the air being $40°C$, what will be the temperature of the ball after 40 minutes from the original?	**07**
	c	Find the general solution of the equation $(px - y)(py + x) = a^2p$ by reducing into Clairaut's form, taking the substitution $X = x^2$, $Y = y^2$	**07**
Module-4			
Q. 07	a	Solve $\frac{d^3y}{dx^3} - 6\frac{d^2y}{dx^2} + 11\frac{dy}{dx} - 6y = 2e^{3x} + 3$	**06**
	b	Solve $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 1 + 3x + x^2$	**07**
	c	Using method of variation of parameters, solve $\frac{d^2y}{dx^2} + a^2y = \tan ax$	**07**
OR			
Q. 08	a	Solve $\frac{d^3y}{dx^3} + 4\frac{dy}{dx} = \cos 2x$	**06**
	b	Solve $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = \sin h(2x + 3)$	**07**
	c	Solve $x^2\frac{d^2y}{dx^2} - 3x\frac{dy}{dx} + 4y = (1 + x)^2$	**07**
Module-5			
Q. 09	a	Find the rank of the matrix $\begin{bmatrix} 2 & 1 & -1 & 3 \\ 1 & 2 & 4 & 3 \\ 3 & 6 & 12 & 9 \\ 3 & 3 & 3 & 6 \end{bmatrix}$	**06**

	b	Solve the system of equations by using the Gauss-Jordan method $x + y + z = 10,$ $2x - y + 3z = 19,$ $x + 2y + 3z = 22$	**07**
	c	Using Rayleigh's power method find the dominant eigenvalue and the corresponding eigenvector of $\begin{bmatrix} 4 & 1 & -1 \\ 2 & 3 & -1 \\ -2 & 1 & 5 \end{bmatrix}$ by taking $[1 \quad 0 \quad 0]^T$ as initial eigen vector [carry out 6 iterations]	**07**
		OR	
Q. 10	a	Find the rank of the matrix $\begin{bmatrix} 11 & 12 & 13 & 14 \\ 12 & 13 & 14 & 15 \\ 13 & 14 & 15 & 16 \\ 14 & 15 & 16 & 17 \end{bmatrix}$	06
	b	For what values λ and μ the system of equations $x + y + z = 6; \quad x + 2y + 3z = 10; \quad x + 2y + \lambda z = \mu,$ has (i) no solution (ii) a unique solution and (iii) infinite number of solutions	**07**
	c	Solve the system of equations $2x - 3y + 20z = 25;$ $20x + y - 2z = 17;$ $3x + 20y - z = -18,$ Using the Gauss-Seidel method, taking (0, 0, 0) as an initial approximate root (Carry out 4 iterations).	**07**

Table showing the Bloom's Taxonomy Level, Course Outcome and Program Outcome

Question		Bloom's Taxonomy Level attached	Course Outcome	Program Outcome
Q.1	(a)	L1	CO 01	PO 01
	(b)	L2	CO 01	PO 01
	(c)	L3	CO 01	PO 02
Q.2	(a)	L1	CO 01	PO 01
	(b)	L2	CO 01	PO 01
	(c)	L3	CO 01	PO 02
Q.3	(a)	L2	CO 02	PO 01
	(b)	L2	CO 02	PO 01
	(c)	L3	CO 02	PO 03

Q.4	(a)	L2	CO 02	PO 01
	(b)	L2	CO 02	PO 01
	(c)	L3	CO 02	PO 02
Q.5	(a)	L2	CO 03	PO 02
	(b)	L3	CO 03	PO 03
	(c)	L2	CO 03	PO 01
Q.6	(a)	L2	CO 03	PO 02
	(b)	L3	CO 03	PO 03
	(c)	L2	CO 03	PO 01
Q.7	(a)	L2	CO 04	PO 01
	(b)	L2	CO 04	PO 01
	(c)	L2	CO 04	PO 02
Q.8	(a)	L2	CO 04	PO 01
	(b)	L2	CO 04	PO 01
	(c)	L2	CO 04	PO 02
Q.9	(a)	L2	CO 05	PO 01
	(b)	L3	CO 05	PO 01
	(c)	L3	CO 05	PO 02
Q.10	(a)	L2	CO 05	PO 01
	(b)	L3	CO 05	PO 02
	(c)	L3	CO 05	PO 01

Bloom's Taxonomy Levels	**Lower order thinking skills**		
	Remembering (Knowledge): L_1	Understanding (Comprehension): L_2	Applying (Application): L_3
	Higher-order thinking skills		
	Analyzing (Analysis): L_4	Valuating (Evaluation): L_5	Creating (Synthesis): L_6