Model Question Paper-2 with effect from 2019-20 (CBCS Scheme)

USN ☐☐☐☐☐☐☐☐☐☐   **18MAT41**

Fourth Semester B.E. Degree Examination
Complex Analysis, Probability and Statistical Methods
(Common to all Programmes)

Time: 3 Hrs   Max.Marks: 100

Note: Answer any FIVE full questions, choosing at least ONE question from each module.
Use of statistical tables allowed.

Module-1

1 (a) Show that $w = f(z) = \log z$ $(z \neq 0)$ is analytic, using Cauchy-Riemann equations. **(06 Marks)**

   (b) Derive Cauchy-Riemann equations in polar form. **(07Marks)**

   (c) Find the analytic function $f(z) = u + iv$, given $u + v = x + y + e^x(\cos y + \sin y)$. **(07Marks)**

OR

2 (a) Show that real and imaginary parts of an analytic function are harmonic. **(06 Marks)**

   (b) If $f(z)$ is a regular function of z, show that $\{(\partial/\partial x)|f(z)|\}^2 + \{(\partial/\partial y)|f(z)|\}^2 = |f'(z)|^2$. **(07Marks)**

   (c) If $w = f(z) = \varphi(x, y) + i\psi(x, y)$ represents the complex potential of an electrostatic field, where **(07Marks)**
        $\psi = (x^2 - y^2) + [x/(x^2 + y^2)]$, find $f(z)$ and hence determine φ.

Module-2

3 (a) State and prove Cauchy's integral formula. **(06 Marks)**

   (b) Discuss the transformation $w = f(z) = e^z$ with respect to the lines parallel to the co-ordinate axes in **(07Marks)**
        z-plane.

   (c) Find the bilinear transformation which maps the points $z = \infty, i, 0$ into the points $w = -1, -i, 1$ **(07Marks)**
        respectively. Also, find the fixed points of the transformation.

OR

4 (a) Verify Cauchy's theorem for $\int_c \frac{1}{z} dz$ where c is the triangle with the vertices (1, 2), (3,2) and (1,4). **(06 Marks)**

   (b) Evaluate: $\int_c \frac{dz}{z^2-4}$ over the curve $C: |z + 2| = 1$. **(07Marks)**

   (c) Find the bilinear transformation which maps   the points $z = 1, i, -1$ into the points $w = 2, i, -2$ **(07Marks)**
        respectively. What are the invariant points under this transformation?

Module-3

5 (a) **(06 Marks)**

The probability distribution of a random variable X is given by the following table:

$X(= x_i)$	0	1	2	3	4	5
$P(X)$	k	$5k$	$10k$	$10k$	$5k$	k

Find (i) the value of k (ii) $P(x \leq 1)$ (iii)$P(0 \leq x < 3)$.

(b) In a certain city, the duration of the shower is exponentially distributed with mean 5 minutes. What is the probability that a shower will last for (i) 10 minutes or more (ii) less than 10 minutes (iii) between 10 to 12 minutes? **(07Marks)**

(c) The marks of 1000 students in an examination follow a normal distribution with mean 70 and standard deviation 5. Find the number of the students whose marks will be (i) less than 65 (ii) more than 75 (iii) between 65 and 75. **(07Marks)**

OR

6 (a) The probability density function of a random variable $X(= x)$ is $f(x) = \begin{cases} kx^2, & -3 < x < 3 \\ 0, & otherwise \end{cases}$. Find (i) the value of k (ii) $P(1 \leq x \leq 2)$ and (iii) $P(x > 1)$. **(06 Marks)**

(b) Suppose 300 misprints are randomly distributed throughout a book of 500 pages, find the probability that a given page contains (i) exactly three misprints (ii) less than three misprints and (iii) four or more misprints. **(07Marks)**

(c) The I.Q. of students in a certain college is assumed to be normally distributed with mean 100 and variance 25. If two students are selected at random, find the probability that (i) both of them have I.Q. between 102 and 110 (ii) at least one of them have I.Q .between 102 and 110 (iii) at most one of them have I.Q. between 102 and 110. **(07Marks)**

Module-4

7 (a) Ten competitors in a beauty contest are ranked by two judges A and B in the following order: **(06 Marks)**

ID No. of competitors	1	2	3	4	5	6	7	8	9	10
Judge A	1	6	5	3	10	2	4	9	7	8
Judge B	6	4	9	8	1	2	3	10	5	7

Calculate the rank correlation coefficient

(b) Fit a best fitting parabola $y = ax^2 + bx + c$ for the following data: **(07Marks)**

x	1	2	3	4	5	6	7	8	9
y	2	6	7	8	10	11	11	10	9

(c) With usual notation, compute $\bar{x}, \bar{y}$ and r from the following lines of regression: **(07Marks)**
$2x + 3y + 1 = 0$ and $x + 6y - 4 = 0$.

OR

8 (a) If θ is the acute angle between the lines of regression, then show that $\tan\theta = \frac{\sigma_x \sigma_y}{\sigma_x^2 + \sigma_y^2}\left(\frac{1-r^2}{r}\right)$. Explain the **(06 Marks)**
significance when $r = 0$ & $r = \pm 1$.

(b) The following table gives the heights of fathers(x) and sons (y): **(07Marks)**

x	65	66	67	67	68	69	70	72
y	67	68	65	68	72	72	69	71

Calculate the coefficient of correlation and lines of regression.

(c) A simply supported beam carries a concentrated load P at its midpoint. Corresponding to various values **(07Marks)**
of P, the maximum deflection Y is measured and is given below:

P	100	120	140	160	180	200
Y	0.45	0.55	0.55	0.70	0.80	0.85

Find a best fitting straight line in the form $Y = a + bP$ to the above data and hence estimate
Y when $P = 150$.

Module-5

9 (a) The joint probability distribution of desecrate random variables X and Y is given below: **(06 Marks)**

X \ Y	1	3	6
1	1/9	1/6	1/18
3	1/6	1/4	1/12
6	1/18	1/12	1/36

Determine (i) marginal distribution of X and Y (ii) Are X and Y statistically independent?

(b) A sample of 100 students is taken from a large population. The mean height of the students in this **(07Marks)**
sample is 160cm. Can it be reasonably regarded that in the population the mean height is 165cm and
the standard deviation is 10cm at 5% level of significance?.

(c) A random sample of 10 boys had the following I.Q. : $70, 120, 110, 101, 88, 83, 95, 98, 107, 100$. Does **(07Marks)**
this data support the assumption of a population mean I.Q. of 100 at 5% level of significance? ($t_{0.05}$ for
9 d.f. is 2.262).

OR

10 (a) Explain the terms: (i) Null hypothesis (ii) Confidence intervals (iii) Type-I and Type-II errors   **(06 Marks)**

   (b) A stenographer claims that she can type at the rate of 120 words per minute. Can we reject her claim on   **(07Marks)**
       the basis of 100 trails in which she demonstrates a mean of 116 words with a standard deviation of 15
       words? Use 5% level of significance.

   (c) Four coins are tossed 100 times and the following results were obtained: **(07Marks)**

No. of Heads	0	1	2	3	4
Frequency	5	29	36	25	5

   Fit a binomial distribution for the data and test the goodness of fit ($\chi^2_{0.05}$ for 4 d.f. is 9.49)**.**