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# Fourth Semester B.E.(CBCS) Examination <br> Additional Mathematics - II 

(Common to all Branches)
Time: 3 Hrs
Max.Marks: 80
Note: Answer any FIVE full questions, choosing at least ONE question from each module.

## Module-I

1. (a) Find the rank of the matrix $\left[\begin{array}{cccc}0 & 1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2\end{array}\right]$ by elementary applying row transformations.
(06 Marks)
(b) Solve the following system of linear equations by Gauss elimination method:
$x+2 y+z=3 ; 2 x+3 y+3 z=10 ; 3 x-y+2 z=13$.
(05 Marks)
(c) Find the inverse of the matrix $\left[\begin{array}{rr}5 & -2 \\ 3 & 1\end{array}\right]$ using Cayley-Hamilton theorem.

## OR

2. (a) Find all the eigenvalues and eigenvector corresponding the smallest eigenvalue of $\left[\begin{array}{rrr}1 & -3 & 2 \\ 4 & 4 & -1 \\ 6 & 3 & 5\end{array}\right]$
(06 Marks)
(b) Reduce the matrix $\left[\begin{array}{ccc}3 & -1 & 2 \\ 1 & 0 & 4 \\ 3 & 1 & 2\end{array}\right]$ into its echelon form and hence find its rank.
(05 Marks)
(c) Solve the system of linear equations $x+y+z=9 ; 2 x-3 y+4 z=13 ; 3 x+4 y+5 z=40$ by applying Gauss elimination method.
(05 Marks)

## Module-II

3. (a) Solve: $\left(D^{2}+1\right) y=\operatorname{cosec} x$ by the method of variation of parameters.
(b) ) Solve: $\left(D^{3}-1\right) y=3 \cos 2 x$
(c) ) Solve: $\left(D^{3}+2 D^{2}+D\right) y=x^{3}$

## OR

4. (a) Solve: $\left(D^{2}-1\right) y=8 x e^{x}$ by the method of undetermined coefficients.
(b) Solve: $\left(D^{3}-7 D+6\right) y=1-x+x^{2}$, where $D=d / d x$
(c) ) Solve: $\left(D^{2}-2 D+5\right) y=e^{2 x} \sin x$

## Module-III

5. (a) Find the Laplace transforms of (i) $t^{2} \cos 3 t$ (ii) $\left(1-e^{-a t}\right) / t$
(b) Find (i) $L\{3 \sqrt{t}+4 / \sqrt{t}\}$ (ii) $L\{\cos t \cos 2 t \cos 3 t\}$
(06 Marks)
(c) Find the Laplace transform of $f(t)=\left\{\begin{array}{l}E, \quad 0 \leq t \leq a / 2 \\ -E, a / 2 \leq t \leq a\end{array}\right.$ where $f(t+a)=f(t)$.

## OR

6. (a) Find the Laplace transforms of (i) $[\sqrt{t}+1 / \sqrt{t}]^{3}$ (ii) $e^{3 t} \sin 5 t \sin 3 t$
(c) Express $f(t)=\left\{\begin{array}{ll}\sin t & 0 \leq t \leq \pi \\ \cos t, & t>\pi\end{array} \quad\right.$ in terms of unit step function and hence find $L\{f(t)\}$.
(05 Marks)

## Module-IV

7. (a) Using Laplace transforms, solve $\frac{d^{2} y}{d t^{2}}+2 \frac{d y}{d t}+2 y=5 \sin t$ subject to the initial conditions $y(0)=0=y^{\prime}(0)$
(06 Marks)
(a) Find the inverse Laplace transforms of (i) $L^{-1}\left\{(s+2)^{3} / s^{6}\right\}$ (ii) $L^{-1}\left\{(s+5) /\left(s^{2}-6 s+13\right)\right\}$
(c) Find (i) $L^{-1}[\log \{(s+a) /(s+b)\}]$ (ii) $L^{-1}\left\{3 s+2 /\left(s^{2}-s-2\right)\right\}$
(05 Marks)

## OR

8. (a) By applying Laplace transforms, solve $\frac{d^{2} y}{d x^{2}}+4 \frac{d y}{d x}+3 y=e^{-t}$ subject to the initial conditions $y(0)=1=y^{\prime}(0)$.
(06 Marks)
(a) Find the inverse Laplace transforms of (i) $L^{-1}\left\{3 s+5 \sqrt{5} /\left(s^{2}+8\right)\right\}$ (ii) $L^{-1}\left\{(2 s-1) /\left(s^{2}+4 s+29\right)\right\}$
(c) Find (i) $L^{-1}\left[\cot ^{-1}(s / a)\right]$ (ii) $L^{-1}\left[4 s+5 /\left\{(s+2)(s+1)^{2}\right\}\right]$
(05 Marks)

## Module-V

9. (a) State the axiomatic definition of probability. For any two arbitrary events $A$ and $B$, prove that $\mathrm{P}(A \cup B)=\mathrm{P}(A)+\mathrm{P}(B)-\mathrm{P}(A \cap B)$.
(06 Marks)
(b) The probability that a team wins a match is $3 / 5$. If this team play 3 matches in a tournament, what is the probability that the team (i) win \& (ii) loose, all the matches.
(05 Marks)
(c) In an UG class of a reputed engineering college, $70 \%$ are boys and $30 \%$ are girls; $5 \%$ of boys and $3 \%$ of the girls are irregular to the classes. What is the probability of a student selected at random is irregular to the classes and what is the probability that the irregular student is a girl?
(05 Marks)

## OR

10. (a) State and prove Bayes's theorem.
(06 Marks)
(b) If A and B are independent events, show that the events $\bar{A}$ and $\bar{B}$ are also independent.
(c) A pair of dice is tossed. Find the probability of scoring " 7 " points?
