17ME43: Applied Thermodynamics
Module 1: Gas Power Cycles

Syllabus:

- Air standard power cycles.
- Description, efficiency and mean effective pressure of
  - Carnot cycle.
  - Otto cycle.
  - Diesel cycle.
  - Dual cycle.
  - Stirling cycle.
- Comparison of Otto and Diesel cycles.
- Description and analysis of gas turbine (Brayton) cycle.
- Description and analysis of
  - Regenerative gas turbine cycle.
  - Inter-cooling and reheating in gas turbine cycles.
- Jet propulsion
  - Introduction to the principles of jet propulsion.
  - Classification of gas turbine engines for jet propulsion.
    - Ideal turbojet cycle.
    - Turbo-prop (prop-jet) engine.
    - Turbo-fan (fan-jet or by-pass turbojet)) engine.
    - Ram-jet engine (Lorin tube).
    - Pulse-jet engine.

Textbooks for reference:

1. Thermodynamics: An Engineering Approach, Yunus A Cengel, Michael A Boles, McGraw Hill Education (India) Private Limited, Eighth edition in SI Units, 2016.
2. Basic and Applied Thermodynamics, P.K. Nag, McGraw Hill Education (India) Private Limited, Second edition, 2010.

Learning Outcomes:

On completion of this module, students will be able to

1. Name the various power cycles and jet propulsion systems; and describe their working principles.
2. Explain and interrelate the various thermodynamic processes involved in power cycles.
3. Draw/sketch the p-v and T-s diagrams of the power cycles.
4. Compare Otto and Diesel power cycles.
5. Develop and derive the expression for efficiency and mean effective pressure of power cycles.
6. Evaluate the performance of power cycles.

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1.1 Introduction

Power cycles are those thermodynamic cycles in which the heat energy is partly absorbed to produce net mechanical work.

The power cycles are classified into

(i) **Gas power cycles** – use gas as the working fluid and the gas does not undergo change of phase.
Ex: IC Engines, Gas Turbines.

(ii) **Vapour power cycles** – use vapour as the working substance and the vapour undergoes phase change.
Ex: Steam Engines, Steam Turbines.

1.2 Air standard power cycles

The power cycles which assume air as the working substance are known as air standard power cycles.

The following assumptions are made for the analysis of air standard cycles:

1) The air of fixed mass undergoes a cyclic process.
2) The air behaves like a perfect gas.
3) Specific heats of air remain constant during the cycle.
4) Thermodynamic processes of the cycle are reversible.
5) The combustion process is replaced by a heat-addition process from an external source.
6) The exhaust process is replaced by a heat-rejection process which brings the state of air to its initial state, thus completing the cycle.
7) The compression and expansion processes undergo reversible adiabatic (isentropic) processes.

1.3 Terminology related to reciprocating engines

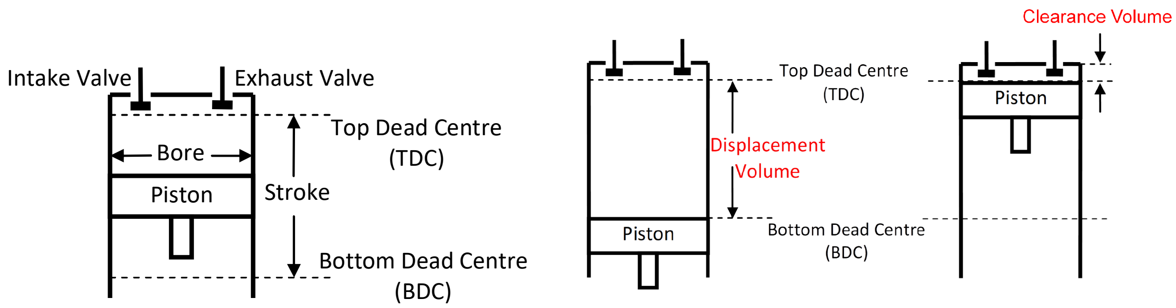


Fig. 1.1 Terminology related to reciprocating engine

Some important terminologies related to reciprocating engine are shown in Fig. 1.1 and are explained as follows:

Bore: It is the diameter of the piston.

Top dead centre (TDC): It is the position of the piston when it forms the smallest volume in the cylinder.

Bottom dead centre (BDC): It is the position of the piston when it forms the largest volume in the cylinder.

Stroke: It is the distance between the top dead centre (TDC) and bottom dead centre (BDC) that the piston can travel in one direction.

Intake valve: It allows the air or air-fuel mixture to be drawn into the cylinder.

Exhaust valve: It allows the combustion products to be expelled from the cylinder.

Clearance volume: The Clearance Volume is the minimum volume formed in the cylinder when the piston is at the Top Dead Centre (TDC).

Displacement volume: The Displacement Volume (or Stroke Volume) is the volume equal to the difference in total volume and clearance volume of the cycle.

Swept volume: When the piston moves from TDC to BDC, the volume displaced by the piston is called Swept Volume

Compression ratio: The Compression Ratio r_c of an engine is the ratio of the maximum volume to the minimum volume formed in the cylinder.

Mean effective pressure:

The mean effective pressure (mep or p_m or MEP) is a fictitious pressure that, if it operated on the piston during the entire power stroke, would produce the same amount of net work as that produced during the actual cycle. The concept of mean effective pressure is illustrated in Fig. 1.2.

$$p_m \ or\ MEP = \frac{w_{net}}{v_{\max} - v_{\min}} = \frac{w_{net}}{v_{total} - v_c}$$

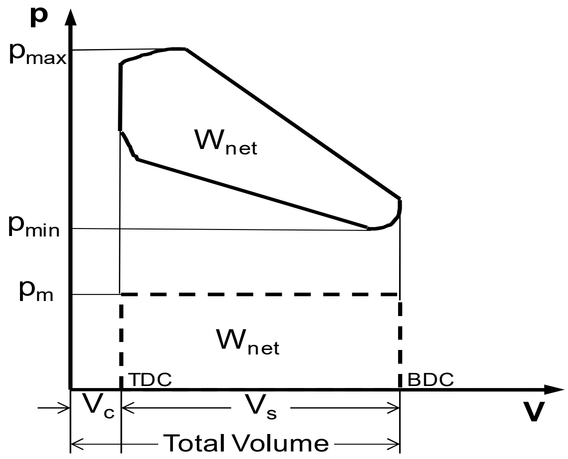


Fig. 1.2 Concept of mean effective pressure

1.4 Carnot Cycle

Carnot cycle is a totally reversible cycle consisting of two isothermal and two isentropic (reversible adiabatic) processes. Carnot cycle can be executed in a closed system or a steady-flow device. In this cycle, either a gas or a vpour can be used as the working fluid. The Carnot cycle is the most efficient cycle that can work between a heat source at temperature T_1 and a sink at temperature T_3. The Carnot cycle cannot be built in reality because it is not a practical cycle. However, the Carnot cycle is used as a standard against which the actual or ideal cycles can be compared. It was invented by Nicolas Leonard Sadi Carnot in the year 1824.

1.4.1 Working of Carnot cycle

The thermodynamic processes that a Carnot cycle undergoes in a closed system (piston-cylinder device) are illustrated in Fig. 1.3. The corresponding p-V and T-s diagrams are shown in Fig. 1.4.

Process 1-2: Isothermal (constant temperature) heat supply

In this process, heat Q_{1-2} is supplied to the working fluid (air or any gas) isothermally at temperature T_1 from an external heat source. The piston moves from position 1 to position 2, increasing the volume from V_1 to V_2. The pressure decreases from p_1 to p_2.

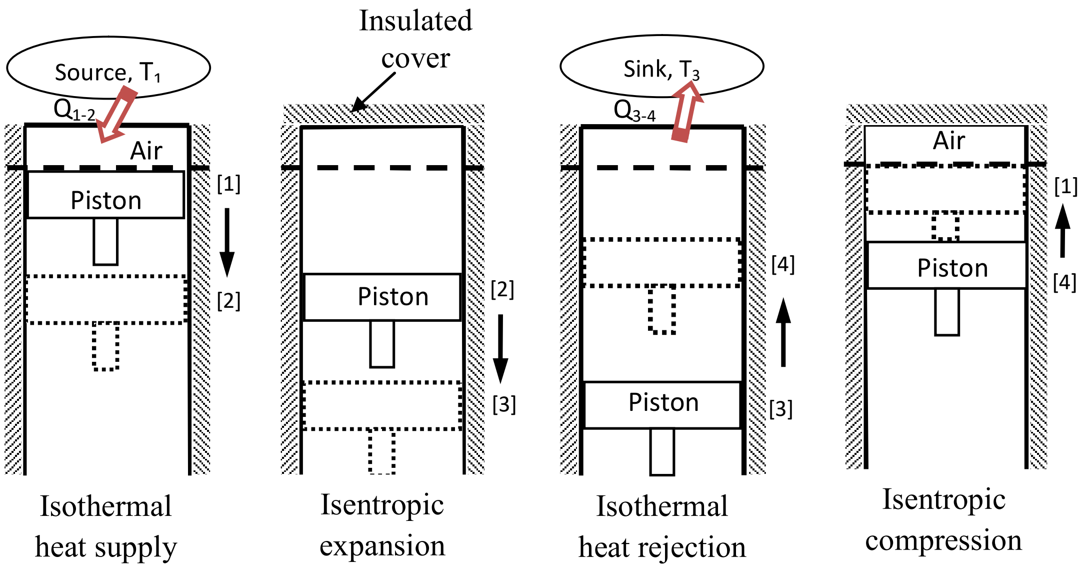


Fig. 1.3 Thermodynamic processes of Carnot cycle

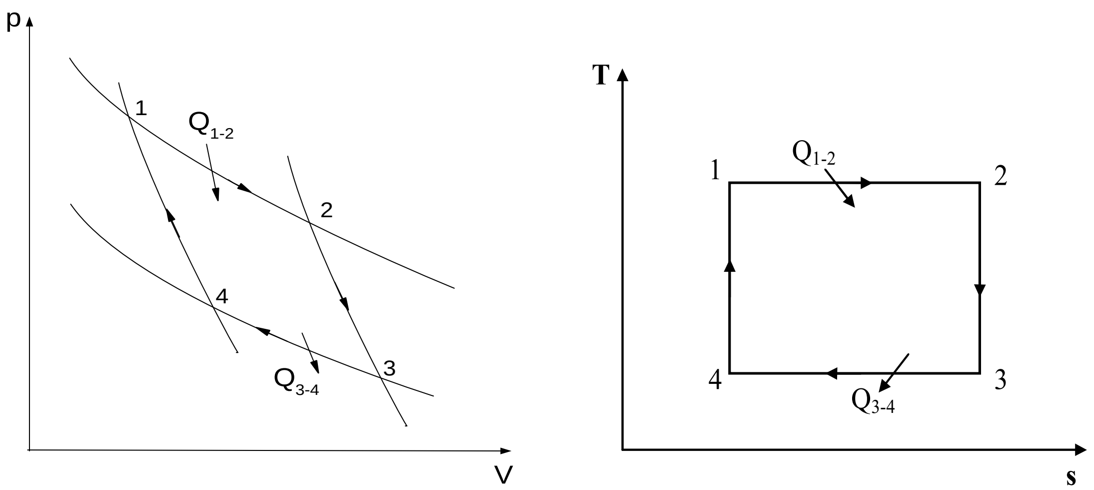


Fig. 1.4 p-V and T-s diagrams of Carnot cycle

Process 2-3: Isentropic (reversible adiabatic) expansion

During this process, the cylinder is completely insulated and the working fluid is allowed to expand isentropically utilizing its own internal energy. The pressure and temperature of the working fluid decreases to p_3 and T_3 respectively. The piston reaches the bottom dead centre (BDC) at the end of this process.

Process 3-4: Isothermal (constant temperature) heat rejected

The working fluid rejects the heat, $Q_{3\text{-}4}$, isothermally at a constant temperature T_3 to an external sink as the piston starts moving towards the top dead centre (TDC). The pressure increases to p_4 and volume decreases to v_4.

Process 4-1: Isentropic (reversible adiabatic) compression

During this process, the working fluid is compressed isentropically to the initial state p_1, V_1 and T_1. The work is done on the working fluid and internal energy increases. The cycle is completed at the end of this process.

1.4.2 Thermodynamic Analysis of Carnot cycle

Referring to the p-V and T-s diagrams shown in Fig. 1.4,

Heat supplied: $Q_{\sup} = Q_{1-2} = RT_1 \ln \frac{V_2}{V_1}$

Heat rejected: $Q_{rej} = Q_{3-4} = RT_3 \ln \frac{V_3}{V_4}$

For the process 2-3 (Isentropic expansion),

$$\frac{V_2}{V_3} = \left(\frac{T_3}{T_1}\right)^{1/\gamma-1} ; \frac{V_1}{V_4} = \left(\frac{T_3}{T_1}\right)^{1/\gamma-1} \Rightarrow \frac{V_2}{V_3} = \frac{V_1}{V_4} \; or \; \frac{V_2}{V_1} = \frac{V_3}{V_4}$$

$$\eta = 1 - \frac{Q_{rej}}{Q_{\sup}} = 1 - \frac{RT_3 \ln \frac{V_2}{V_1}}{RT_1 \ln \frac{V_3}{V_4}}$$

$$\therefore \eta = 1 - \frac{T_3}{T_1}$$

1.4.3 Limitations of Carnot Cycle

1) In practice, it is not possible to devise isothermal heat transfer processes. It requires very large heat exchangers and the piston movement be restricted to infinitesimally small steps (quasi-static).

2) The processes are not reversible in reality.

3) The compression and expansion processes undergo **reversible adiabatic** (isentropic) processes, which require the piston to move rapidly so that there is no time for the heat transfer to occur.

1.5 Stirling Cycle

Stirling cycle consists of two reversible isothermal and two constant volume (isochoric) processes. It is a totally reversible cycle, like Carnot cycle, as the heat-addition and heat-rejection processes take place isothermally during the cycle.

1.5.1 Working of Stirling cycle

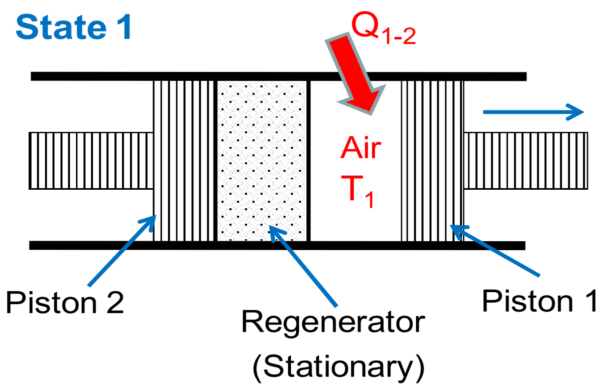


Fig. 1.5 Schematic of Stirling cycle showing state1 of the working fluid

The working of Stirling cycle in a closed system is explained with the help of a schematic diagram shown in Fig. 1.5. The system consists of a cylinder with two pistons on each side and a regenerator in the middle. The regenerator acts as a temporary storage of thermal storage. All the thermodynamic processes of Stirling cycle are shown on p-V and T-s planes in Fig. 1.6.

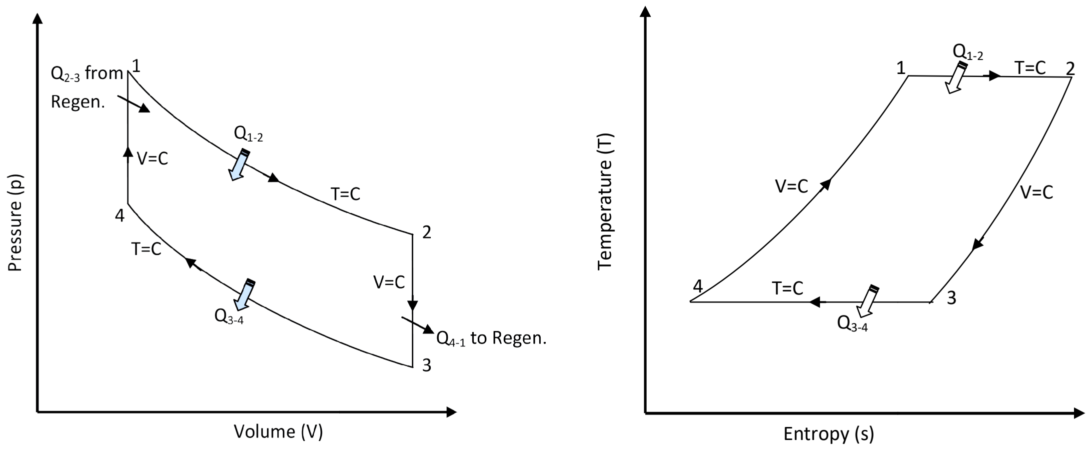


Fig. 1.6 p-V and T-s diagrams of Stirling cycle

State 1: Initially, the right chamber houses the entire working fluid (air or any gas), which is at a high temperature and pressure.

Process 1-2: Isothermal heat addition from the external source

The heat is transferred to the gas at T_1 from a source at T_1. As the gas expands isothermally, the right piston (piston 1) moves outward, doing work and the gas pressure drops.

Process 2-3: Isochoric regeneration

Both pistons (piston 1 and piston 2) are moved to the left at the same rate (so as to keep the volume constant) until the entire gas is forced into the left chamber, passing through the regenerator. The heat is transferred to the regenerator during the passage of gas and the temperature of the gas in the left chamber drops to T_3.Thus, at the state 3, the temperature of the regenerator will be T_1 to the right end and will be T_3 at the left end of the regenerator.

Process 3-4: Isothermal heat rejection to the external sink

The left piston (piston 2) is moved inward, compressing the gas. Heat is transferred from the gas to a sink at temperature T_3 so that the gas temperature remains constant at T_3 while the pressure increases.

Process 4-1: Isochoric regeneration

Both pistons (piston 1 and piston 2) are moved to the right (so as to keep the volume constant), forcing the entire gas to the right chamber. The gas temperature increases from T_3 to T_1 as it passes through the regenerator and picks up the thermal energy stored in the regenerator during process 2-3. This completes the cycle.

1.5.2 Thermodynamic Analysis of Stirling Cycle

Referring to the p-V and T-s diagrams shown in Fig. 1.6,

Heat supplied: $Q_{\sup} = Q_{1-2} = RT_1 \ln \frac{v_2}{v_1}$

Heat rejected: $Q_{rej} = Q_{3-4} = RT_3 \ln \frac{v_3}{v_4}$

From p-v diagram, $\frac{v_2}{v_1} = \frac{v_3}{v_4}$

$$\eta = 1 - \frac{Q_{rej}}{Q_{\sup}} = 1 - \frac{T_3}{T_1}$$

It is observed from the above expression that efficiency of the Stirling cycle is the same as that of Carnot cycle.

1.5.3 Features of Stirling Cycle

1) It is an altered version of Carnot cycle that comprises of two constant volume regeneration processes.

2) The processes are not reversible in reality.

3) The weight to power ratio is high.

4) However, because of their higher efficiency potential and better emission control, Stirling engines with modifications have been tried to compete with the petrol or diesel engines.

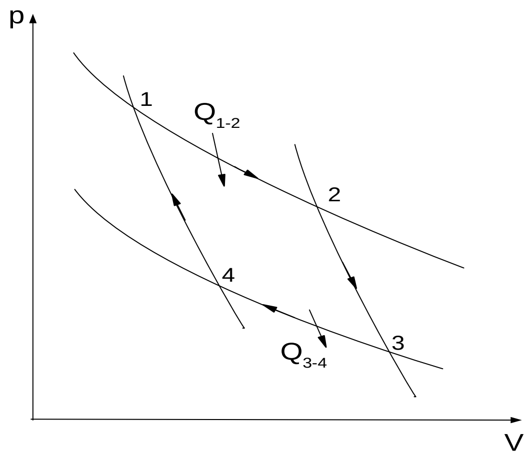
1.6 Numerical examples on Carnot and Stirling cycle

Numerical example 1.6.1

The minimum pressure and temperature of the air standard Carnot cycle are 1 bar and 15^{0}C, respectively. The pressure after isothermal compression is 3.5 bar and the pressure after isentropic compression is 10.5 bar. Determine (i) efficiency (ii) mean effective pressure and (iii) power developed if the Carnot engine makes 2 cycles per second. Take for air R = 0.287 kJ/kgK and γ = 1.4.

Data given:

- $p_3 = 1 \times 10^5$ N/m^2
- $T_3 = 15 + 273 = 288$ K
- $p_4 = 3.5 \times 10^5$ N/m^2
- $p_1 = 10.5 \times 10^5$ N/m^2
- n = 2 cycles/s
- $R = 0.287 \times 10^3$ J/kgK
- $\gamma = 1.4$



p-V diagram

To determine:

(i) Efficiency

(ii) Mean effective pressure

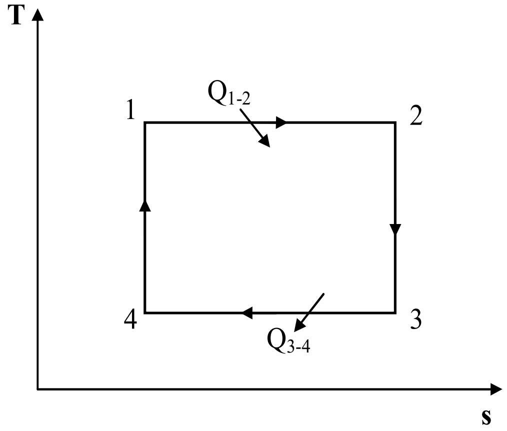
(iii) Power developed

Solution:

At State 3:

$$p_3V_3 = mRT_3 \Rightarrow V_3 = \frac{mRT_3}{p_3}$$

$$\therefore V_3 = \frac{1\times 0.287\times 10^3 \times 288}{1\times 10^5} = 0.826\,m^3$$



T-s diagram

At State 4: $T_4 = T_3 = 288$ K

$$p_4V_4 = mRT_4 \Rightarrow V_4 = \frac{mRT_4}{p_4}$$

$$\therefore V_4 = \frac{1\times 0.287\times 10^3 \times 288}{3.5\times 10^5} = 0.236\,m^3$$

For the process 4-1 (Isentropic or reversible adiabatic):

$$\frac{T_1}{T_4} = \left(\frac{p_1}{p_4}\right)^{\frac{\gamma-1}{\gamma}} \Rightarrow T_1 = 288\times\left(\frac{10.5}{3.5}\right)^{\frac{1.4-1}{1.4}} = 394.19K$$

At State 1:

$$p_1V_1 = mRT_1 \Rightarrow V_1 = \frac{mRT_1}{p_1}$$

$$\therefore V_1 = \frac{1\times 0.287\times 10^3 \times 394.19}{10.5\times 10^5} = 0.108\,m^3$$

For the process 2-3 (Isentropic or reversible adiabatic):

$$\frac{p_2}{p_3} = \left(\frac{T_2}{T_3}\right)^{\frac{\gamma}{\gamma-1}} \Rightarrow p_2 = 1\times 10^5 \times \left(\frac{394.19}{288}\right)^{\frac{1.4}{1.4-1}}$$

$$\therefore p_2 = 3\times 10^5\, N/m^2$$

At State 2:

$$p_2V_2 = mRT_2 \Rightarrow V_2 = \frac{mRT_2}{p_2}$$

$$\therefore V_2 = \frac{1\times 0.287\times 10^3 \times 394.19}{3\times 10^5} = 0.377\,m^3$$

Heat supplied:

$$Q_{1-2} = p_1V_1 \ln\frac{V_2}{V_1} = 10.5\times10^5 \times \ln\frac{0.377}{0.108} = 141.763\times10^3 J$$

Heat rejected:

$$Q_{3-4} = p_3V_3 \ln\frac{V_3}{V_4} = 1\times10^5 \times 0.826\times \ln\frac{0.826}{0.236} = 103.478\times10^3 J$$

Carnot efficiency:

$$\eta = 1-\frac{T_3}{T_1} = 1-\frac{288}{394.19} = 0.2694(26.94\%)$$

Net work done:

$$W = Q_{1-2} - Q_{3-4} = \left(141.763-103.478\right)\times10^3$$

$$\therefore W = 38.285\times10^3 J$$

Power developed:

$$P = W\times n = 38.285\times10^3\times 2 = 76.57\times10^3 J/s(W)$$

Stroke volume:

$$V_s = V_3 - V_1 = 0.826-0.1077 = 0.7183 m^3$$

Mean effective pressure:

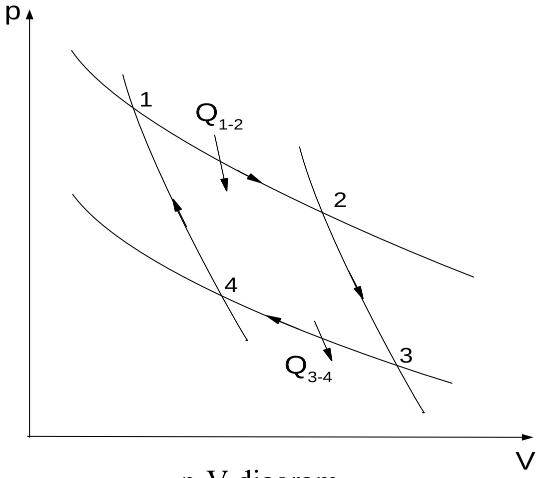
$$p_m = \frac{W}{V_s} = \frac{38.285\times10^3}{0.7183} = 53.299\times10^3 N/m^2$$

Numerical Example 1.6.2

Carnot cycle containing 0.5 kg of air has a thermal efficiency of 50%. The heat transferred to the air during isothermal expansion is 40 kJ. The pressure and volume at the beginning of isothermal expansion are 7 bar and 0.12 m^3 respectively. Calculate (i) the maximum and minimum temperature for the cycle in Kelvin (ii) the volume at the end of isothermal expansion in m^3 (iii) the heat transfer for each of the four processes.

Data given:

- Carnot Cycle
- $m = 0.5$ kg
- $\eta = 0.50$
- $Q_{1-2} = 40 \times 10^3$ J
- $p_1 = 7$ bar $= 7 \times 10^5$ N/m^2
- $V_1 = 0.12$ m^3



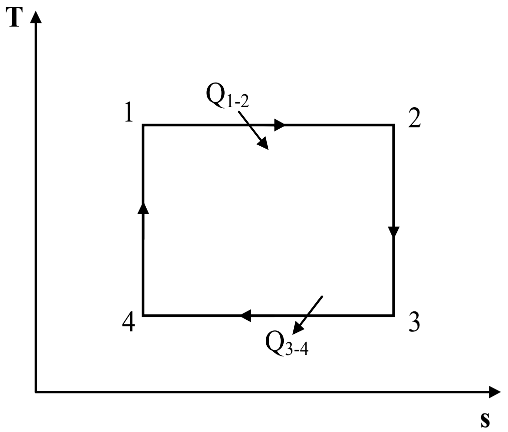
p-V diagram

To calculate:

(i) T_{max} (T_1), T_{min} (T_3)

(ii) V_2

(iii) Q_{1-2} , Q_{2-3} , Q_{3-4} , and Q_{4-1} .



T-s diagram

Solution:

Carnot efficiency:

$$\eta = 1 - \frac{T_3}{T_1} \Rightarrow 0.5 = 1 - \frac{T_3}{585.37}$$

$$\therefore T_3 = 292.69K(19.69^0 C)$$

At State 1:

$$p_1 V_1 = mRT_1 \Rightarrow T_1 = \frac{p_1 V_1}{mR} = \frac{7 \times 10^5 \times 0.12}{0.5 \times 0.287 \times 10^3}$$

$$\therefore T_1 = 585.37K\left(312.37^0 C\right)$$

For the process 1-2 (isothermal):

$$Q_{1-2} = p_1 V_1 \ln \frac{V_2}{V_1} \Rightarrow 40 \times 10^3 = 7 \times 10^5 \times 0.12 \times \ln \frac{V_2}{0.12}$$

$$\therefore V_2 = 0.192 m^3$$

Also, Carnot efficiency:

$$\eta = 1 - \frac{Q_{rej}}{Q_{sup}} \Rightarrow 0.5 = 1 - \frac{Q_{rej}}{40 \times 10^3}$$

$$\therefore Q_{rej} = Q_{3-4} = 20 \times 10^3 J$$

For the processes 2-3 and 4-1 (Isentropic or reversible adiabatic):

$$Q_{2-3} = Q_{4-1} = 0$$

Numerical example 1.6.3

The following data refers to an ideal Stirling cycle with ideal regenerator. Pressure, temperature and volume of the working medium at the beginning of the isothermal compression are 100 kPa, 30^0C and 0.05 m^3 respectively. The clearance volume of the cycle is $1/10^{th}$ the initial volume. The maximum temperature attained in the cycle is 700^0C. Draw p-V and T-s diagrams. Calculate (i) net work done and (ii) thermal efficiency with 100% regenerator efficiency.

Data given:

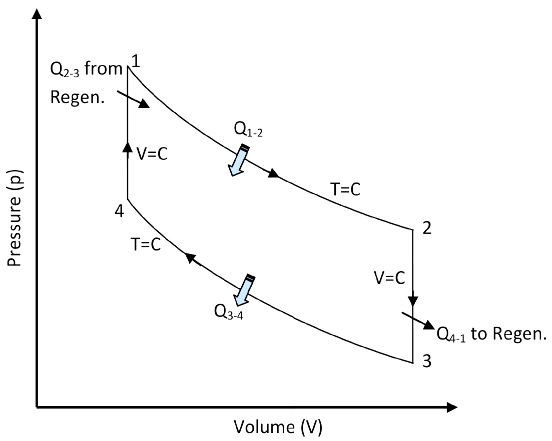
$p_3 = 100 \times 10^3$ N/m^2

$T_3 = 30+273 = 303$ K

$V_3 = 0.05$ m^3 $= V_2$

$V_1 = 0.1\ V_3 = 0.005$ m^3 $= V_4$

$T_1 = 700+273 = 973$ K



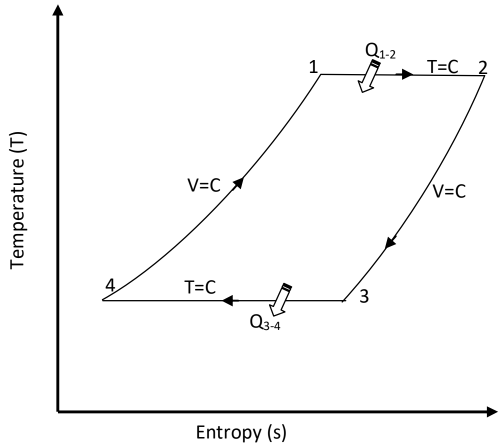
p-V diagram

To determine:

(i) Net work done

(ii) Thermal efficiency

Solution:



T-s diagram

Thermal efficiency:

$$\eta = 1 - \frac{T_3}{T_1} = 1 - \frac{303}{973} = 0.6886(68.86\%)$$

Net work done:

(This can be calculated by any of the following approaches):

Approach 1:

At State 3:

$$p_3V_3 = mRT_3 \Rightarrow m = \frac{p_3V_3}{RT_3} = \frac{100\times10^3\times0.05}{0.287\times10^3\times303}$$

$$\therefore m = 0.057kg$$

For the process 1-2 (isothermal):

Heat supplied:

$$Q_{1-2} = mRT_1\ln\frac{V_2}{V_1} = 0.057\times0.287\times10^3\times973\times\ln\frac{0.05}{0.005}$$

$$\therefore Q_{1-2} = 36.65\times10^3 J$$

For the process 3-4 (isothermal):

Heat rejected:

$$Q_{3-4} = mRT_3\ln\frac{V_3}{V_4} = 0.057\times0.287\times10^3\times303\times\ln\frac{0.05}{0.005}$$

$$\therefore Q_{3-4} = 11.41\times10^3 J$$

Net work done:

$$W = Q_{1-2} - Q_{3-4} = (36.65 - 11.41)\times10^3 = 25.24\times10^3 J$$

Approach 2:

Determine p_2 from Isochoric process 2-3:

$$\frac{p_2}{T_{2=1}} = \frac{p_3}{T_3} \Rightarrow p_2 = \frac{p_3 \times T_{2=1}}{T_3}$$

Calculate the heat supplied:

$$Q_{1-2} = p_2 V_2 \ln \frac{V_2}{V_1}$$

Calculate the heat rejected:

$$Q_{3-4} = p_3 V_3 \ln \frac{V_3}{V_4}$$

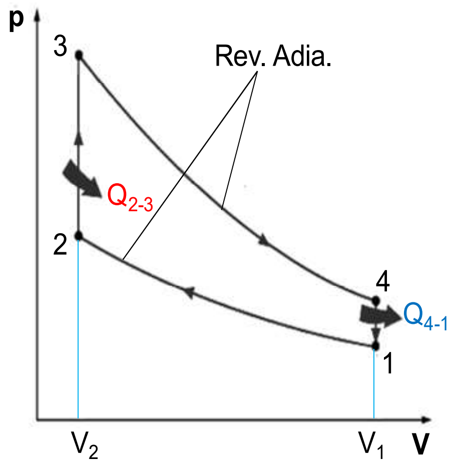
Calculate the net work done:

$$W = Q_{1-2} - Q_{3-4}$$

1.7 Otto Cycle

Otto cycle consists of two isentropic (reversible adiabatic) and two isochoric (reversible constant volume) processes. The cycle was proposed by Frenchman Beau de Rochas in 1876 in Germany. Otto cycle is named after Nikolaus A Otto, who built a successful four-stroke engine in 1876 in Germany. It is the ideal cycle for spark-ignition reciprocating engines.

1.7.1 Working of Otto cycle



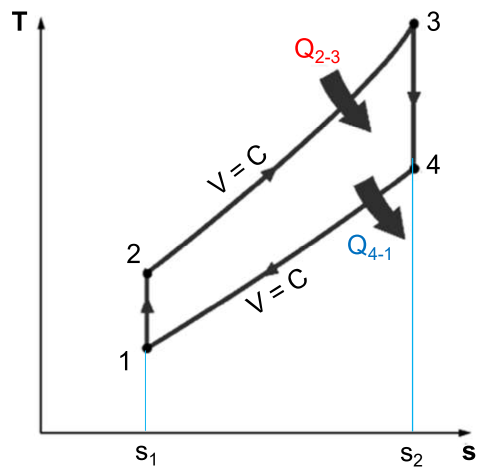


Fig. 1.7 p-V and T-s diagrams of Otto cycle

The various thermodynamic processes that the Otto cycle undergoes are explained with reference to the p-V and T-s diagrams shown in Fig. 1.7.

Process 1-2: Isentropic (reversible adiabatic) compression

Air is compressed isentropically from its initial pressure p_1 and volume v_1 to p_2 and v_2 respectively. The temperature increases from T_1 to T_2.

Process 2-3: Isochoric (reversible constant volume) heat addition

At the end of the isentropic compression, heat is supplied to the air from an external source at constant volume. The temperature of the air rises to T_3 and the pressure increases to p_3.

Process 3-4: Isentropic (reversible adiabatic) expansion

The high pressure and temperature is now allowed to expand isentropically thus doing the work. The pressure and temperature decreases to p_4 and T_4 respectively.

Process 4-1: Isochoric (reversible constant volume) heat rejection

At the end of the isentropic expansion, the heat is rejected by the air thus attaining the initial pressure and temperature p_1 and T_1 respectively. This completes the Otto cycle.

1.7.2 Thermodynamic Analysis of Otto cycle

Referring to the p-V and T-s diagrams shown in Fig. 1.7,

Heat supplied: $Q_{sup} = Q_{2-3} = mC_v\left(T_3 - T_2\right)$

Heat rejected: $Q_{rej} = Q_{4-1} = mC_v\left(T_4 - T_1\right)$

$$\eta = 1 - \frac{Q_{rej}}{Q_{sup}} = 1 - \frac{mC_v\left(T_4 - T_1\right)}{mC_v\left(T_3 - T_2\right)} = 1 - \frac{\left(T_4 - T_1\right)}{\left(T_3 - T_2\right)}$$

$$\Rightarrow \eta = 1 - \frac{T_1\left(T_4/T_1 - 1\right)}{T_2\left(T_3/T_2 - 1\right)}$$

As the processes 1-2 and 3-4 are isentropic;

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma-1} \quad and \quad \frac{T_3}{T_4} = \left(\frac{V_4}{V_3}\right)^{\gamma-1}$$

As $V_3 = V_2$ and $V_4 = V_1$; $\frac{V_1}{V_2} = \frac{V_4}{V_3}$

$$\Rightarrow \frac{T_2}{T_1} = \frac{T_3}{T_4} \quad or \quad \frac{T_4}{T_1} = \frac{T_3}{T_2}$$

$$\Rightarrow \eta_{Otto} = 1 - \frac{T_1}{T_2}$$

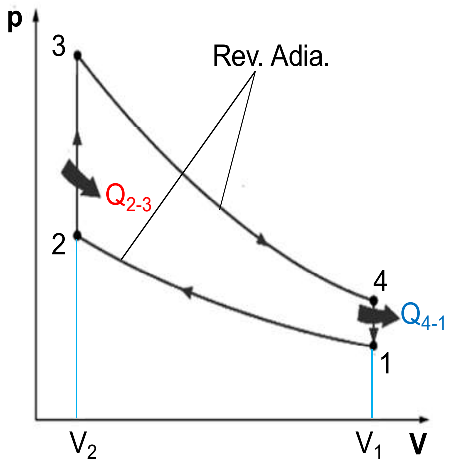
As the process1-2 is isentropic;

$$\frac{T_1}{T_2} = \left(\frac{V_2}{V_1}\right)^{\gamma-1} = \left(\frac{1}{r_c}\right)^{\gamma-1} = \frac{1}{r_c^{\gamma-1}}$$ where the compression ratio is $r_c = V_1/V_2$

$$\therefore \eta_{Otto} = 1 - \frac{1}{r_c^{\gamma-1}}$$

1.7.3 Mean Effective Pressure (p_m or mep)

$$p_m(or\ mep) = \frac{W_{net}}{V_s} = \frac{Q_{net}}{V_1 - V_2} = \frac{Q_{sup} - Q_{rej}}{V_1\left[1-\left(V_2/V_1\right)\right]}$$



p-V diagram

Where

$$Q_{sup} = Q_{2-3} = mC_v\left(T_3 - T_2\right)$$

$$Q_{rej} = Q_{4-1} = mC_v\left(T_4 - T_1\right)$$

$$V_1 = \frac{mRT_1}{p_1} \quad and \quad \frac{V_2}{V_1} = \frac{1}{r_c}$$

On substitution into Eqn. (1);

$$p_m = \frac{\cancel{m}C_v\left(T_3 - T_2\right) - \cancel{m}C_v\left(T_4 - T_1\right)}{\left(\cancel{m}RT_1/p_1\right)\left[1-\left(1/r_c\right)\right]}$$

$$p_m = \frac{\cancel{C_v}\left(T_3 - T_2\right) - \cancel{C_v}\left(T_4 - T_1\right)}{\left[\underbrace{\cancel{C_v}\left(\gamma-1\right)}_{R}T_1 \Big/ p_1\right]\left(r_c - 1\right)/r_c}$$

$$R = C_p - C_v \Rightarrow \frac{R}{C_v} = \frac{C_p}{C_v} - 1 = \gamma - 1$$

$$\therefore R = C_v\left(\gamma - 1\right)$$

$$p_m = \frac{p_1 r_c\left\{\left[T_2\left(\left(T_3/T_2\right)-1\right)\right] - T_1\left[\left(T_4/T_1\right)-1\right]\right\}}{T_1\left(\gamma-1\right)\left(r_c - 1\right)} \quad \text{------ (2)}$$

As the processes 1-2 and 3-4 are isentropic,

and $V_3 = V_2$, $V_4 = V_1$

$$\frac{T_4}{T_1} = \frac{T_3}{T_2} \ and \ T_2 = T_1\ r_c^{\gamma-1}$$

$$\Rightarrow \frac{T_2}{T_1} = \frac{T_3}{T_4}\ ;\ or\ \frac{T_2}{T_1} = \frac{T_3}{T_4} = \left(\frac{V_4}{V_3}\right)^{\gamma-1} = r_c^{\gamma-1} \Rightarrow T_2 = T_1 r_c^{\gamma-1}$$

As the process 2-3 is isochoric;

$$\frac{p_2}{T_2} = \frac{p_3}{T_3} \Rightarrow \frac{T_3}{T_2} = \frac{p_3}{p_2} = \alpha \ (Explosion\ Ratio)$$

On substitution into Eqn. (2),

$$p_m = \frac{p_1 r_c \left\{\left[\cancel{T_1} r_c^{\gamma-1}(\alpha-1)\right] - \cancel{T_1}[\alpha-1]\right\}}{\cancel{T_1}(\gamma-1)(r_c-1)}$$

$$p_m = \frac{p_1 r_c (\alpha-1)\left(r_c^{\gamma-1}-1\right)}{(\gamma-1)(r_c-1)}$$

1.8 Numerical examples on Otto cycle

Numerical example 1.8.1

An engine working on the Otto cycle is supplied with air at 0.1 MPa, 35^{0}C. The compression ratio is 8. Heat supplied is 2100 kJ/kg. Calculate the maximum pressure and temperature of the cycle, the cycle efficiency, and the mean effective pressure. (For air, c_p = 1.005kJ/kgK, c_v = 0.718 kJ/kgK, and R = 0.287 kJ/kgK)

Data given:

- Otto Cycle
- p_1 = 0.1 MPa = 0.1 x 10^6 N/m^2
- T_1 = 35^{0}C = 35 + 273 = 308 K
- r_c = 8
- $q_{in} = q_{2\text{-}3}$ = 2100 kJ/kg = 2100 x 10^3 J/kgK
- c_p = 2100 kJ/kg = 2100 x 10^3 J/kgK
- c_p = 1.005 kJ/kgK = 1.005 x 10^3 J/kgK
- c_v = 0.718 kJ/kgK = 0.718 x 10^3 J/kgK
- R = 0.287 kJ/kgK = 0.287 x 10^3 J/kgK

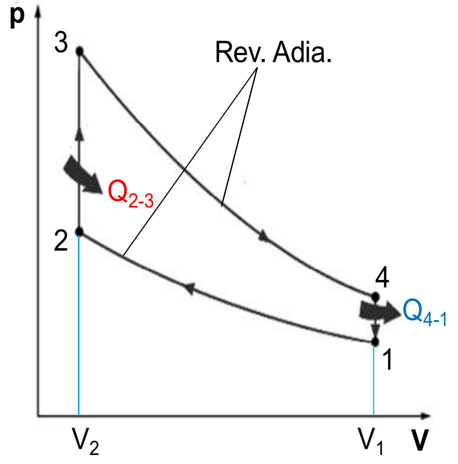
To calculate:

(i) Maximum pressure (p_3)

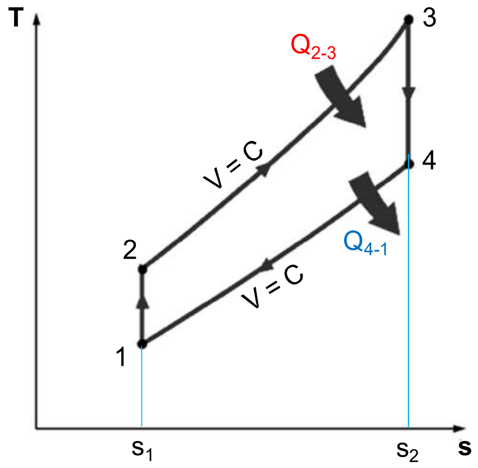
(ii) Maximum temperature (T_3)

(iii) Cycle efficiency (η)

(iv) MEP (p_m)



p-V diagram



T-s diagram

Solution:

Given that: $r_c = \frac{v_1}{v_2} = 8$

Cycle efficiency:

$$\eta = 1 - \frac{1}{r_c^{\gamma-1}} = 1 - \frac{1}{8^{1.4-1}} = 0.5647(56.47\%)$$

Also, Cycle efficiency:

$$\eta = \frac{w_{net}}{q_{2-3}} \Rightarrow w_{net} = \eta \times q_{2-3}$$

$$\therefore w_{net} = 0.5647 \times 2100 \times 10^3 = 1185.87 \times 10^3 J/kg$$

At State 1:

$$p_1 v_1 = RT_1 \Rightarrow v_1 = \frac{RT_1}{p_1} = \frac{0.287 \times 10^3 \times 308}{0.1 \times 10^6}$$

$$\therefore v_1 = 0.88 m^3/kg$$

Given that:

$$r_c = \frac{v_1}{v_2} = 8 \Rightarrow v_2 = \frac{v_1}{8} = \frac{0.88}{8}$$

$$\therefore v_2 = 0.11 m^3/kg$$

Mean effective pressure:

$$p_m = \frac{w_{net}}{v_1 - v_2} = \frac{1185.87 \times 10^3}{0.88 - 0.11} = 1540.09 \times 10^3 N/m^2$$

Given that: c_p = 1.005 kJ/kgK, c_v = 0.718 kJ/kgK

$$\gamma = \frac{c_p}{c_v} = \frac{1.005}{0.718} = 1.4$$

For isentropic process 1-2:

$$\frac{T_2}{T_1} = \left(\frac{v_1}{v_2}\right)^{\gamma-1} = \left(r_c\right)^{\gamma-1} \Rightarrow T_2 = T_1 r_c^{\gamma-1}$$
$$\therefore T_2 = 308 \times 8^{1.4-1} = 707.6K(434.6^0 C)$$

and

$$\frac{p_2}{p_1} = \left(\frac{v_1}{v_2}\right)^{\gamma} = \left(r_c\right)^{\gamma} \Rightarrow p_2 = p_1 r_c^{\gamma}$$
$$\therefore p_2 = 0.1 \times 10^6 \times 8^{1.4} = 1.84 \times 10^6 N/m^2$$

Given that: $q_{2\text{-}3}$ = 2100 x 10^3 J/kg

$$q_{2-3} = c_v\left(T_3 - T_2\right) \Rightarrow 2100 \times 10^3 = 0.718 \times 10^3\left(T_3 - 707.6\right)$$
$$\therefore T_3 = 3632.39K(3359.39^0 C)$$

For constant volume process 2-3:

$$\frac{p_3}{p_2} = \frac{T_3}{T_2} \Rightarrow p_3 = p_2\left(\frac{T_3}{T_2}\right) = 1.84 \times 10^6\left(\frac{3632.39}{707.6}\right)$$
$$\therefore p_3 = 9.46 \times 10^6 N/m^2$$

Numerical example 1.8.2

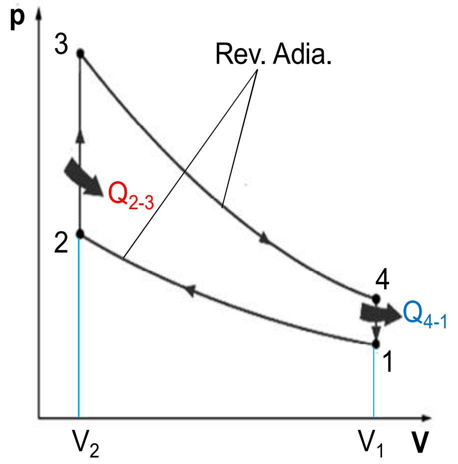
An engine of 250 mm bore and 375 mm stroke works on constant volume cycle. The clearance volume is 0.00263 m^3. The initial pressure and temperature are 1 bar and 50^0C. If the maximum pressure is 25 bar, determine (i) the air standard efficiency of the cycle and (ii) the mean effective pressure.

Data given:

Constant Volume Cycle (Otto Cycle)

- d = 250 mm = 250 x 10^{-3} m

- $L = 375\ mm = 375 \times 10^{-3}\ m$
- $v_2 = v_3 = 0.00263\ m^3$
- $p_1 = 1\ bar = 1 \times 10^5\ N/m^2$
- $T_1 = 50^0C = 50 + 273 = 323\ K$
- $p_3 = 25\ bar = 25 \times 10^5\ N/m^2$



p-V diagram

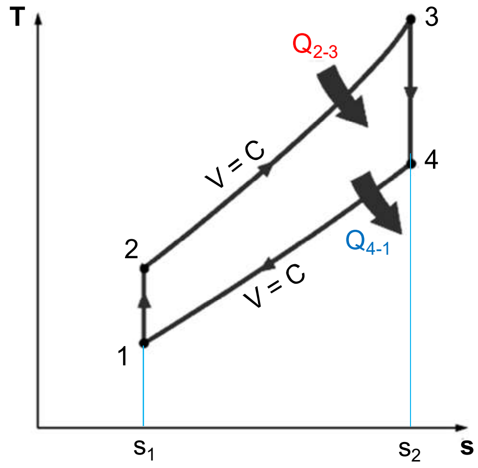
For air, assume that:

- $c_p = 1.005 kJ/kgK = 1.005 \times 10^3\ J/kgK$
- $c_v = 0.718\ kJ/kgK = 0.718 \times 10^3\ J/kgK$
- $R = 0.287\ kJ/kgK = 0.287 \times 10^3\ J/kgK$
- $\gamma = 1.4$

To determine:

(i) Air standard efficiency (η_{Otto})

(ii) Mean effective pressure (p_m)



T-s diagram

Solution:

Stroke volume, V_s:

$$V_s = \frac{\pi d^2 L}{4} = \frac{\pi \times \left(250 \times 10^{-3}\right)^2 \times 375 \times 10^{-3}}{4} = 0.0184 m^3$$

Total volume, V:

$$V = V_c + V_s = 0.00263 + 0.0184 = 0.02103 m^3$$

Compression ratio, r_c:

$$r_c = \frac{V_1}{V_2} = \frac{0.02103}{0.00263} = 7.996$$

Air standard efficiency, η:

$$\eta = 1 - \frac{1}{r_c^{\gamma-1}} = 1 - \frac{1}{7.996^{1.4-1}} = 0.5646(56.46\%)$$

For isentropic process 1-2:

$$\frac{p_2}{p_1} = \left(\frac{v_1}{v_2}\right)^{\gamma} = (r_c)^{\gamma} \Rightarrow p_2 = p_1 r_c^{\gamma}$$

$$\therefore p_2 = 1 \times 10^5 \times 7.996^{1.4} = 18.37 \times 10^5 \, N/m^2$$

Given that: $p_3 = 25 \times 10^5$ N/m^2

Explosion ratio, α:

$$\alpha = \frac{p_3}{p_2} = \frac{25}{18.37} = 1.36$$

Mean effective pressure, p_m:

$$p_m = \frac{p_1 r_c (\alpha - 1)\left(r_c^{\gamma-1} - 1\right)}{(\gamma - 1)(r_c - 1)}$$

$$p_m = \frac{1 \times 10^5 \times 7.996(1.36 - 1)\left(7.996^{1.4-1} - 1\right)}{(1.4 - 1)(7.996 - 1)}$$

$$\therefore p_m = 1.334 \times 10^5 \, N/m^2$$

Numerical example 1.8.3

In an Otto cycle, the upper and lower limits for the absolute temperature are T_3 and T_1, respectively. Prove that the intermediate temperatures are equal and their value is given by

$$\sqrt{T_1 T_3}$$

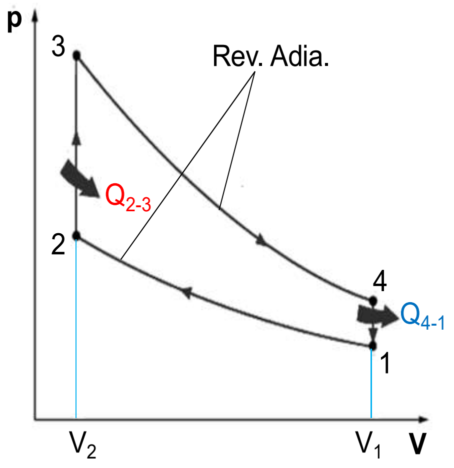
Proof:

For isentropic process 1-2:

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma-1} = r_c^{\gamma-1} \Rightarrow T_2 = T_1 r_c^{\gamma-1}$$

For isentropic process 3-4:

$$\frac{T_4}{T_3}=\left(\frac{V_3}{V_4}\right)^{\gamma-1}=\left(\frac{V_2}{V_1}\right)^{\gamma-1}=\left(\frac{1}{r_c^{\gamma-1}}\right)\Rightarrow T_4=\frac{T_3}{r_c^{\gamma-1}}$$



p-V diagram

Net work done:

$$W=Q_{2-3}-Q_{4-1}=c_v\left(T_3-T_2\right)-c_v\left(T_4-T_1\right)$$

$$\Rightarrow W=c_v\left[\left(T_3-T_1r_c^{\gamma-1}\right)-\left(T_3r_c^{1-\gamma}-T_1\right)\right]$$

In the above equation, T_1 , T_3 , γ and c_v are fixed.

For maximum work: $\frac{dW}{dr_c}=0$

$$\frac{d\left\{c_v\left[\left(T_3-T_1r_c^{\gamma-1}\right)-\left(T_3r_c^{1-\gamma}-T_1\right)\right]\right\}}{dr_c}=0$$

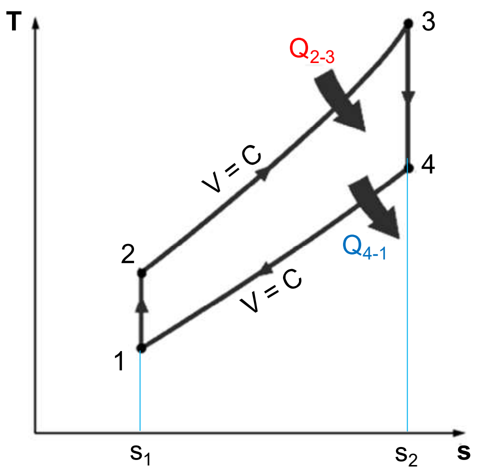
Differentiating with respect to r_c and equating to zero:

$$-T_1\left(\gamma-1\right)r_c^{\gamma-1-1}-T_3\left(1-\gamma\right)r_c^{1-\gamma-1}=0$$

$$-T_1\left(\gamma-1\right)r_c^{\gamma-2}+T_3\left(\gamma-1\right)r_c^{-\gamma}=0$$

$$T_1r_c^{\gamma-2}=T_3r_c^{-\gamma}$$

$$\frac{T_3}{T_1}=\frac{r_c^{\gamma-2}}{r_c^{-\gamma}}=r_c^{2\gamma-2}=r_c^{2(\gamma-1)}\Rightarrow r_c=\left(\frac{T_3}{T_1}\right)^{\frac{1}{2(\gamma-1)}}$$



T-s diagram

For isentropic process 1-2:

$$T_2=T_1r_c^{\gamma-1}=T_1\left[\left(\frac{T_3}{T_1}\right)^{\frac{1}{2(\gamma-1)}}\right]^{\gamma-1}\Rightarrow T_2=\sqrt{T_1T_3}$$

For isentropic process 3-4:

$$T_4=\frac{T_3}{r_c^{\gamma-1}}=\frac{T_3}{\left[\left(\frac{T_3}{T_1}\right)^{\frac{1}{2(\gamma-1)}}\right]^{\gamma-1}}\Rightarrow T_4=\sqrt{T_1T_3}$$

Thus: $T_2=T_4=\sqrt{T_1T_3}$

1.9 Diesel Cycle

Diesel cycle consists of two isentropic (reversible adiabatic), one isobaric (reversible constant pressure) and one isochoric (reversible constant volume) processes. The cycle was first proposed by Rudolph Diesel in the 1890s. The Diesel cycle is the ideal cycle for compression-ignition engines.

1.9.1 Working of Diesel Cycle

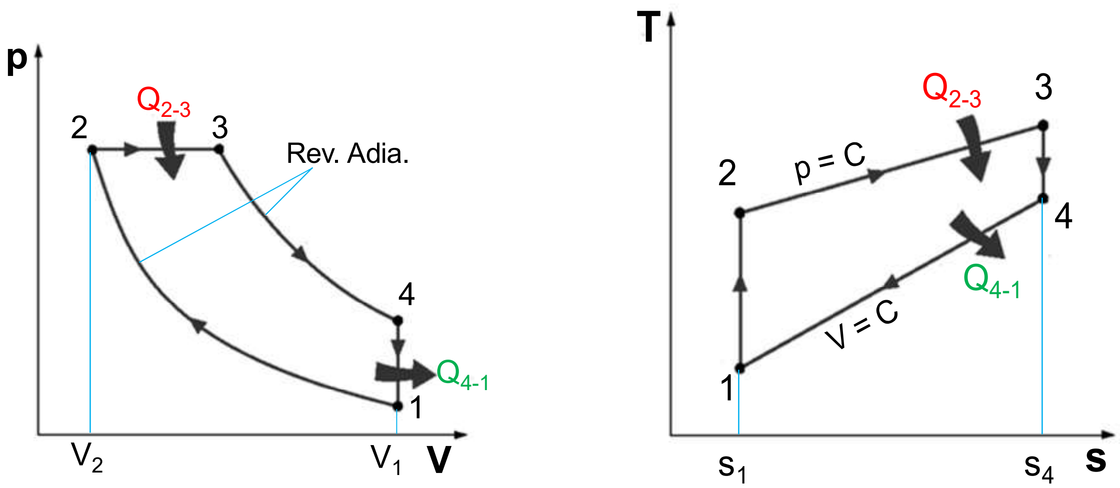


Fig. 1.8 p-V and T-s diagrams of Diesel cycle

The Diesel cycle is plotted on p-v and T-s planes as shown in Fig. 1.8. The working of Diesel cycle is explained as follows:

Process 1-2: Isentropic (reversible adiabatic) compression

During this process, the air from initial state is compressed isentropically to pressure p_2 and temperature T_2. The volume is decreased from v_1 to v_2.

Process 2-3: Isobaric (reversible constant pressure) heat supply

At the end of the isentropic compression, heat is supplied to the air at constant pressure from an external source. The temperature increases from T_2 to T_3. The supply of heat is continued till the point 3 is reached. This point is called ‘cut-off’ point. The volume increases from v_2 to v_3.

Process 3-4: Isentropic (reversible adiabatic) expansion

At point 3, the supply of heat is 'cut off' and the air is allowed to expand doing the work. The pressure and temperature decreases to p_4 and T_4 respectively. The volume increases to v_4.

Process 4-1: Isochoric (reversible constant volume) heat rejection

At the end of isentropic expansion, the heat is rejected from the air to an external sink, thus bringing the pressure and temperature of the air to its initial state. This completes the Diesel cycle.

1.9.2 Thermodynamic analysis of Diesel Cycle

Referring to the p-V and T-s diagrams shown in Fig. 1.8,

Heat supplied: $Q_{\sup} = Q_{2-3} = mC_p\left(T_3 - T_2\right)$

Heat rejected: $Q_{rej} = Q_{4-1} = mC_v\left(T_4 - T_1\right)$

$$\eta = 1 - \frac{Q_{rej}}{Q_{\sup}} = 1 - \frac{mC_v\left(T_4 - T_1\right)}{mC_p\left(T_3 - T_2\right)} = 1 - \frac{\left(T_4 - T_1\right)}{\gamma\left(T_3 - T_2\right)}$$

$$\Rightarrow \eta = 1 - \frac{T_1\left[\left(T_4/T_1\right) - 1\right]}{\gamma T_2\left[\left(T_3/T_2\right) - 1\right]} \qquad \text{------ (3)}$$

Let's define the volume ratios:

Compression ratio: $r_c = \frac{V_1}{V_2}$

Expansion ratio: $r_e = \frac{V_4}{V_3}$

Cut-off ratio: $\rho = \frac{V_3}{V_2}$

$$\Rightarrow r_c = r_e\rho$$

Recalling that;

For process 1-2: Isentropic or reversible adiabatic compression

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma-1} = r_c^{\gamma-1} \Rightarrow \frac{T_2}{T_1} = r_c^{\gamma-1}$$

For process 2-3: Isobaric or constant pressure heat supply

$$\frac{V_2}{T_2} = \frac{V_3}{T_3} \Rightarrow \frac{T_3}{T_2} = \frac{V_3}{V_2} = \rho \Rightarrow \frac{T_3}{T_2} = \rho$$

For process 3-4: Isentropic or reversible adiabatic expansion

$$\frac{T_4}{T_3} = \left(\frac{V_3}{\underbrace{V_4}_{=V_1}}\right)^{\gamma-1} = \left(\frac{V_3/V_2}{V_1/V_2}\right)^{\gamma-1} = \left(\frac{\rho}{r_c}\right)^{\gamma-1}$$

$$\Rightarrow \frac{T_4}{T_1 r_c^{\gamma-1} \rho} = \left(\frac{\rho}{r_c}\right)^{\gamma-1} ;\Rightarrow \frac{T_4}{T_1} = \rho^{\gamma}$$

On substitution into Eqn. (3);

$$\Rightarrow \eta_{Diesel} = 1 - \frac{\left[\rho^{\gamma} - 1\right]}{\gamma\, r_c^{\gamma-1}\left[\rho - 1\right]}$$

$$\therefore \eta_{Diesel} = 1 - \frac{1}{r_c^{\gamma-1}}\left[\frac{\rho^{\gamma} - 1}{\gamma\left(\rho - 1\right)}\right]$$

Note:

For the same r_c, $\eta_{Diesel} < \eta_{Otto}$ because, $\rho > 1$ and $\left[\frac{\rho^{\gamma} - 1}{\gamma\left(\rho - 1\right)}\right] > 1.$

1.9.3 Mean effective pressure (p_m or mep)

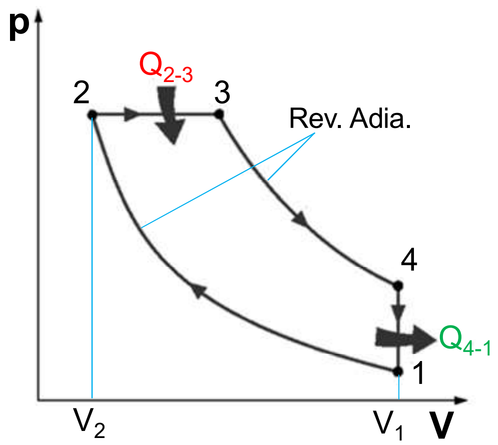
$$p_m(or\ mep) = \frac{W_{net}}{V_s} = \frac{Q_{net}}{V_1 - V_2} = \frac{Q_{sup} - Q_{rej}}{V_1\left[1-\left(V_2/V_1\right)\right]} \qquad \text{------ (4)}$$

where

$$Q_{sup} = Q_{2-3} = mC_p\left(T_3 - T_2\right)$$

$$Q_{rej} = Q_{4-1} = mC_v\left(T_4 - T_1\right)$$

$$V_1 = \frac{mRT_1}{p_1} \quad and \quad \frac{V_2}{V_1} = \frac{1}{r_c}$$



p-V diagram

On substitution into Eqn. (4);

$$p_m = \frac{mC_p\left(T_3-T_2\right) - mC_v\left(T_4-T_1\right)}{\left(mRT_1/p_1\right)\left[1-\left(1/r_c\right)\right]}$$

$$p_m = \frac{C_v\left[\left(C_p/C_v\right)\left(T_3-T_2\right)\right] - C_v\left(T_4-T_1\right)}{\left[\underbrace{C_v\left(\gamma-1\right)}_{R}T_1 \Big/ p_1\right]\left(r_c-1\right)/r_c}$$

$$p_m = \frac{p_1 r_c\left\{\left[\gamma T_2\left(\left(T_3/T_2\right)-1\right)\right] - T_1\left[\left(T_4/T_1\right)-1\right]\right\}}{T_1\left(\gamma-1\right)\left(r_c-1\right)} \qquad \text{------ (5)}$$

Recalling that: $\frac{T_3}{T_2} = \rho \qquad \frac{T_2}{T_1} = r_c^{\gamma-1} \qquad \frac{T_4}{T_1} = \rho^{\gamma}$

On substitution into Eqn. (5);

$$p_m = \frac{p_1 r_c^{\gamma-1}\left\{\left[\gamma\ r_c^{\gamma-1}\left(\rho-1\right)\right] - \left[\left(\rho^{\gamma}-1\right)\right]\right\}}{\left(\gamma-1\right)\left(r_c^{\gamma-1}-1\right)}$$

$$\therefore p_m = \frac{p_1 r_c\left[\gamma\ r_c^{\gamma-1}\left(\rho-1\right) - \left(\rho^{\gamma}-1\right)\right]}{\left(\gamma-1\right)\left(r_c-1\right)}$$

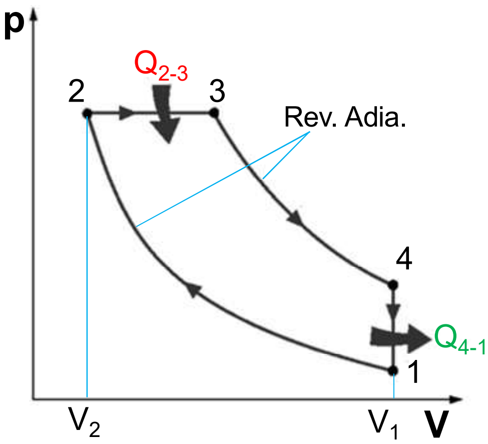
1.10 Numerical examples on Diesel cycle

Numerical example 1.10.1

The compression ratio of an air standard diesel cycle is 16. The temperature and pressure at the beginning of isentropic compression are 15^{0}C and 0.1 MPa respectively. During the constant pressure process, the heat is added until the temperature reaches 1480^{0}C. Determine (i) the cut-off ratio (ii) the heat supplied per kg of air, (iii) the cycle efficiency, and (iv) the m.e.p

Data given:

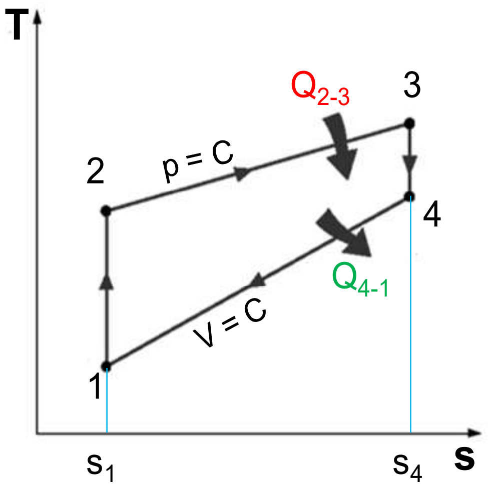
- Diesel Cycle
- $r_c = (v_1 / v_2) = 16$
- $T_1 = 15^0\text{C} = 15 + 273 = 288$ K
- $p_1 = 0.1$ MPa $= 0.1 \times 10^6$ N/m^2
- $T_3 = 1480^0\text{C} = 1480 + 273 = 1753$ K



p-V diagram

For air, assume that

- $c_p = 1.005$kJ/kgK $= 1.005 \times 10^3$ J/kgK
- $c_v = 0.718$ kJ/kgK $= 0.718 \times 10^3$ J/kgK
- $R = 0.287$ kJ/kgK $= 0.287 \times 10^3$ J/kgK
- $\gamma = 1.4$



T-s diagram

To calculate:

(i) Cut-off ratio (v_3/v_2)

(ii) Heat supplied per kg of air (q_{2-3})

(iii) Cycle efficiency (η)

(iv) MEP (p_m)

Solution:

For isentropic process 1-2:

$$\frac{T_2}{T_1}=\left(\frac{v_1}{v_2}\right)^{\gamma-1}=\left(r_c\right)^{\gamma-1}\Rightarrow T_2=T_1 r_c^{\gamma-1}$$

$$\therefore T_2=288\times16^{1.4-1}=873.05K(600.05^0C)$$

For constant pressure process 2-3:

$$Cut-off\ \ ratio:\rho=\frac{V_3}{V_2}=\frac{T_3}{T_2}=\frac{1753}{873.05}=2.01$$

Heat Supplied, q_{2-3}:

$$q_{2-3}=c_p\left(T_3-T_2\right)=1.005\times10^3\times\left(1753-873.05\right)$$

$$\therefore q_{2-3}=884.35\times10^3 J/kg$$

For Isentropic expansion process 3-4:

$$\frac{T_4}{T_3}=\left(\frac{\rho}{r_c}\right)^{\gamma-1}\Rightarrow T_4=T_3\left(\frac{\rho}{r_c}\right)^{\gamma-1}=1753\times\left(\frac{2.01}{16}\right)^{1.4-1}$$

$$\therefore T_4=764.56K(491.56^0C)$$

Heat rejected, q_{4-1}:

$$q_{4-1}=c_v\left(T_4-T_1\right)=0.718\times10^3\times\left(764.56-288\right)$$

$$\therefore q_{4-1}=342.17\times10^3 J/kg$$

Cycle efficiency, η:

$$\eta=1-\frac{q_{4-1}}{q_{2-3}}=1-\frac{342.17}{884.35}=0.6131(61.31\%)$$

At State 1:

$$p_1v_1=RT_1\Rightarrow v_1=\frac{RT_1}{p_1}=\frac{0.287\times10^3\times288}{0.1\times10^6}=0.827m^3/kg$$

Mean effective pressure, p_m:

$$p_m = \frac{w_{net}}{v_s} = \frac{q_{2-3} - q_{4-1}}{v_1 - v_2} = \frac{q_{2-3} - q_{4-1}}{v_1\left(1 - \frac{1}{r_c}\right)} = \frac{884.35 - 342.17}{0.827\left(1 - \frac{1}{16}\right)}$$

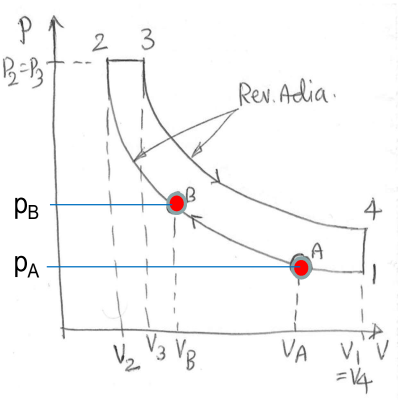
$$\therefore p_m = 6.993 \times 10^5 N/m^2$$

Numerical example 1.10.2

The pressures on the compression curve of a Diesel engine are given as 14 bar at 1/8th of stroke and 1.4 bar at 7/8th of the stroke. Determine (i) the compression ratio (ii) the air standard efficiency and (iii) the mean effective pressure if the cut-off occurs at 1/16th of the stroke. Assume pressure and temperature at the beginning of adiabatic compression as 1 bar and 27⁰C respectively.

Data given:

- Diesel Cycle
- p_A = 1.4 bar = 1.4 x 10^5 N/m^2
- $v_A = v_2 + (7/8)\, v_s$
- p_B = 14 bar = 14 x 10^5 N/m^2
- $v_B = v_2 + (1/8)\, v_s$
- p_1 = 1 bar = 1 x 10^5 N/m^2
- $T_1 = 27^0$C = 27 + 273 = 300 K
- $v_3 = v_2 + (1/16)\, v_s$



p-V diagram

For air, assume that:

- c_p = 1.005kJ/kgK = 1.005 x 10^3 J/kgK
- c_v = 0.718 kJ/kgK = 0.718 x 10^3 J/kgK
- R = 0.287 kJ/kgK = 0.287 x 10^3 J/kgK
- γ = 1.4

To calculate:

(i) Cut-off ratio (v_3/v_2)

(ii) Air standard efficiency (η)

(iii) MEP (p_m)

Solution:

For isentropic compression process *A-B*:

$$\frac{p_A}{p_B} = \left(\frac{v_B}{v_A}\right)^{\gamma} = \left[\frac{v_2 + (1/8)v_s}{v_2 + (7/8)v_s}\right]^{\gamma} = \left[\frac{8v_2 + v_s}{8v_2 + 7v_s}\right]^{\gamma}$$

$$\therefore \frac{1.4}{14} = \left[\frac{8v_2 + v_s}{8v_2 + 7v_s}\right]^{1.4}$$

$$0.1931 = \left[\frac{8v_2 + v_s}{8v_2 + 7v_s}\right]$$

$$\therefore \frac{v_s}{v_2} = 18.35$$

Compression ratio, r_c:

$$r_c = 1 + \frac{v_s}{v_2} = 1 + 18.35 = 19.35$$

Given that:

$$v_3 = v_2 + \left(\frac{1}{16}\right)\left(\frac{v_s}{v_2}\right)$$

$$\therefore \rho = \frac{v_3}{v_2} = 1 + \left(\frac{1}{16}\right)18.35 = 1.14$$

Air standard efficiency, η:

$$\%\eta_{Diesel} = \left\{1 - \frac{1}{r_c^{\gamma-1}}\left[\frac{\rho^{\gamma} - 1}{\gamma(\rho - 1)}\right]\right\} = \left\{1 - \frac{1}{19.35^{1.4-1}}\left[\frac{1.14^{1.4} - 1}{1.4(1.14 - 1)}\right]\right\}$$

$$= 0.686(68.6\%)$$

Mean effective pressure:

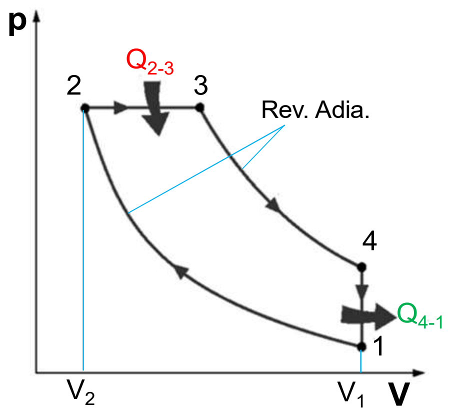
$$p_m = \frac{p_1 r_c\left[\gamma r_c^{\gamma-1}(\rho-1)-\left(\rho^{\gamma}-1\right)\right]}{(\gamma-1)\left(r_c-1\right)}$$

$$= \frac{1\times10^5\times19.35\left[1.4\times19.35^{1.4-1}(1.14-1)-\left(1.14^{1.4}-1\right)\right]}{(1.4-1)(19.35-1)}$$

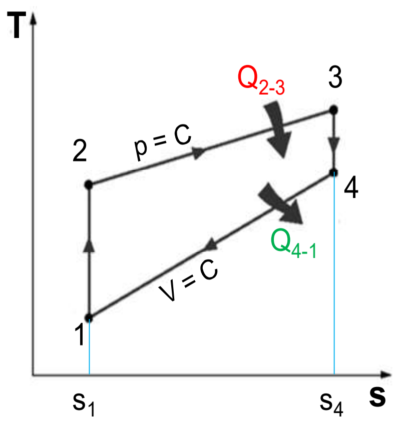
$$p_m = 1.159\times10^5\,N/m^2$$

Numerical example 1.10.3

A diesel cycle has compression ratio of 14 and the cut off ratio of 2.2. Calculate the air standard efficiency and pressure and temperature at all salient points, assuming that the pressure and temperature at the beginning of the cycle as 0.98 bar and 100^{0}C respectively.



p-V diagram



T-s diagram

Data given:

- Diesel Cycle
- r_c = 14
- ρ = 2.2
- p_1 = 0.98 bar = 0.98 x 10^5 N/m^2
- T_1 = 100^{0}C = 100 + 273 = 373 K

For air, assume that:

- c_p = 1.005kJ/kgK = 1.005 x 10^3 J/kgK
- c_v = 0.718 kJ/kgK = 0.718 x 10^3 J/kgK
- R = 0.287 kJ/kgK = 0.287 x 10^3 J/kgK
- γ = 1.4

To calculate:

(i) Air standard efficiency (η)

(ii) Pressure and temperature at all salient points

Solution:

Air standard efficiency:

$$\eta = \left\{1 - \frac{1}{r^{\gamma-1}}\left[\frac{\rho^{\gamma}-1}{\gamma(\rho-1)}\right]\right\} =$$

$$\therefore \eta = \left\{1 - \frac{1}{14^{1.4-1}}\left[\frac{2.2^{1.4}-1}{1.4(2.2-1)}\right]\right\} = 0.5825(58.25\%)$$

For process 1-2 (Isentropic or reversible adiabatic compression)

$$\frac{p_2}{p_1} = \left(\frac{v_1}{v_2}\right)^{\gamma} = (r_c)^{\gamma} \Rightarrow p_2 = p_1 r_c^{\gamma}$$

$$\therefore p_2 = 0.98 \times 10^5 \times 14^{1.4} = 39.43 \times 10^5\, N/m^2$$

and

$$\frac{T_2}{T_1} = \left(\frac{v_1}{v_2}\right)^{\gamma-1} = (r_c)^{\gamma-1} \Rightarrow T_2 = T_1 r_c^{\gamma-1}$$

$$\therefore T_2 = 373 \times 14^{1.4-1} = 1071.91K$$

For process 2-3 (Isobaric or constant pressure heat supply)

$$p_3 = p_2 = 39.43 \times 10^5\, N/m^2$$

$$\frac{V_3}{T_3} = \frac{V_2}{T_2} \Rightarrow \frac{T_3}{T_2} = \frac{V_3}{V_2} = \rho \Rightarrow T_3 = T_2\rho$$

$$\therefore T_3 = 1071.91 \times 2.2 = 2358.20K$$

For process 3-4 (Isentropic or reversible adiabatic expansion)

$$r_e = \frac{V_4}{V_3} = \frac{r_c}{\rho} = \frac{14}{2.2} = 6.36$$

$$\frac{p_4}{p_3} = \left(\frac{V_3}{V_4}\right)^{\gamma} = \left(\frac{\rho}{r_c}\right)^{\gamma} \Rightarrow p_4 = p_3\left(\frac{\rho}{r_c}\right)^{\gamma}$$

$$\therefore p_4 = 39.43 \times 10^5 \times \left(\frac{1}{6.36}\right)^{1.4} = 2.96 \times 10^5\, N/m^2$$

$$\frac{T_4}{T_3} = \left(\frac{V_3}{V_4}\right)^{\gamma-1} = \left(\frac{\rho}{r_c}\right)^{\gamma-1} \Rightarrow T_4 = T_3\left(\frac{\rho}{r_c}\right)^{\gamma-1}$$

$$\therefore T_4 = 2358.20 \times \left(\frac{1}{6.36}\right)^{1.4-1} = 1125.12K$$

1.11 Dual combustion cycle (Limited pressure cycle/Semi-Diesel cycle/Mixed cycle)

In early compression ignition engines the fuel was injected when the piston reached top dead centre and thus combustion lasted well into the expansion stroke. The air standard Diesel cycle thus does not simulate exactly the pressure and volume variation in an actual modern compression ignition engine. In modern engines the fuel is injected before the top dead centre (about 15^{o}). The dual combustion cycle is the closer approximation to the modern compression ignition engine in which some part of the heat is added to the air at constant volume and remainder at constant pressure.

1.11.1 Working of dual combustion cycle

The p-v and T-s diagrams of a dual combustion cycle are shown in Fig. 1.9. The working of the cycle is explained as follows:

Process 1-2: Isentropic (reversible adiabatic) compression

In this process, the air is compressed in the cylinder isentropically from its initial state p_1, v_1 and T_1 to p_2, v_2, and T_2 respectively.

Process 2-3: Isochoric (constant volume) heat supply

At the end of isentropic compression, the heat is added to air at constant volume. This increases the pressure and temperature of air to p_3 and T_3 respectively.

Process 3-4: Isobaric (constant pressure) heat supply

After the point 3 is reached, the heat is added to air at constant pressure till the point 4 is reached. The point 4 is called 'cut-off' point at which the supply of heat to the air is stopped. The pressure and temperature increases to p_4 and T_4 respectively.

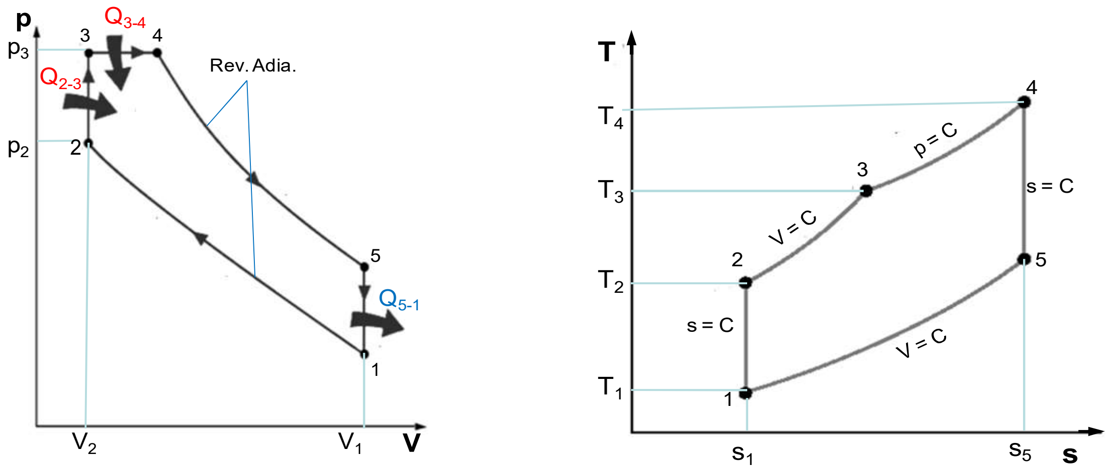


Fig. 1.9 p-V and T-s diagrams of Dual combustion cycle

Process 4-5: Isentropic (reversible adiabatic) expansion

At point 4, the supply of heat is cut off and the air is allowed to expand doing the work. The pressure and temperature decreases to p_5 and T_5 respectively.

Process 5-1: Isochoric (constant volume) heat rejection

At the end of isentropic expansion, the heat is rejected from the air to an external sink and the air is brought back to its initial state. This completes the dual combustion cycle.

1.11.2 Thermodynamic analysis of dual combustion cycle

Referring to the p-V and T-s diagrams shown in Fig. 1.9,

Heat supplied: $Q_{sup} = mC_v\left(T_3 - T_2\right) + mC_p\left(T_4 - T_3\right)$

Heat rejected: $Q_{rej} = mC_v\left(T_5 - T_1\right)$

Efficiency:

$$\eta = 1 - \frac{Q_{out}}{Q_{in}} = 1 - \frac{\cancel{m}C_v\left(T_5 - T_1\right)}{\cancel{m}C_v\left(T_3 - T_2\right) + \cancel{m}C_p\left(T_4 - T_3\right)}$$

$$= 1 - \frac{\left(T_5 - T_1\right)}{\left(T_3 - T_2\right) + \gamma\left(T_4 - T_3\right)}$$

$$\eta = 1 - \frac{T_1\left[\left(T_5/T_1\right)-1\right]}{T_2\left[\left(T_3/T_2\right)-1\right]+\gamma T_3\left[\left(T_4/T_3\right)-1\right]}$$ ------ **(6)**

Let's define the volume ratios:

Compression ratio: $r_c = \frac{V_1}{V_2}$

Cut-off ratio: $\rho = \frac{V_4}{V_3}$

Expansion ratio: $r_e = \frac{V_5}{V_4}$

From the above relationship, we obtain: $r_c = r_e\,\rho$

Explosion ratio: $\alpha = \frac{p_3}{p_2}$

Recalling that:

For process 1-2 (Isentropic or reversible adiabatic compression)

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma-1} = r_c^{\gamma-1} \Rightarrow T_2 = T_1\,r_c^{\gamma-1}$$

For process 2-3: (Isochoric or reversible constant volume heat supply)

$$\frac{T_3}{T_2} = \frac{p_3}{p_2} = \alpha \Rightarrow T_3 = T_2\alpha = T_1\,r_c^{\gamma-1}\alpha$$

For process 3-4 (Isobaric or reversible constant pressure heat supply)

$$\frac{T_4}{T_3} = \frac{V_4}{V_3} = \rho \Rightarrow T_4 = T_3\rho = T_1\,r_c^{\gamma-1}\alpha\rho$$

For process 4-5 (Isentropic or reversible adiabatic expansion)

$$\frac{T_5}{T_4} = \left(\frac{V_4}{\underbrace{V_5}_{=V_1}}\right)^{\gamma-1} = \left(\frac{V_4/V_3}{V_1/\underbrace{V_3}_{=V_2}}\right)^{\gamma-1} = \left(\frac{\rho}{r_c}\right)^{\gamma-1} \Rightarrow T_5 = T_4\left(\frac{\rho}{r_c}\right)^{\gamma-1}$$

$$\Rightarrow T_5 = T_1 r_c^{\gamma-1}\alpha\rho\left(\frac{\rho}{r_c}\right)^{\gamma-1} = T_1\alpha\rho^{\gamma} \Rightarrow \frac{T_5}{T_1} = \alpha\rho^{\gamma}$$

On substitution into Eqn. (6);

$$\eta = 1 - \frac{\cancel{T_1}\left[\alpha\rho^{\gamma}-1\right]}{\cancel{T_1}r_c^{\gamma-1}\left[\alpha-1\right]+\gamma\cancel{T_1}r_c^{\gamma-1}\alpha\left[\rho-1\right]}$$

$$\therefore \eta = 1 - \frac{1}{r_c^{\gamma-1}}\left[\frac{\alpha\rho^{\gamma}-1}{(\alpha-1)+\gamma\alpha(\rho-1)}\right]$$

Observe that:

1. For ρ = 1, we will get expression for efficiency of Otto cycle.
2. For α = 1, we will get expression for efficiency of Diesel cycle.

1.11.3 Mean effective pressure (p_m)

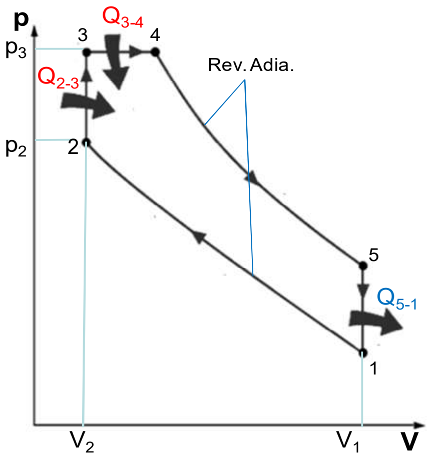
$$p_m = \frac{W_{net}}{V_{max}-V_{min}} = \frac{Q_{net}}{V_1-V_2} = \frac{Q_{in}-Q_{out}}{V_1\left[1-\left(V_2/V_1\right)\right]}$$

$$where \quad Q_{sup} = Q_{2-3}+Q_{3-4} = mC_v\left(T_3-T_2\right)+mC_p\left(T_4-T_3\right)$$

$$and \quad Q_{rej} = Q_{5-1} = mC_v\left(T_5-T_1\right)$$

$$V_1 = \frac{mRT_1}{p_1} \quad and \quad \frac{V_2}{V_1} = \frac{1}{r_c}$$

$$p_m = \frac{\cancel{m}C_v\left(T_3-T_2\right)+\cancel{m}C_p\left(T_4-T_3\right)-\cancel{m}C_v\left(T_5-T_1\right)}{\left(\cancel{m}RT_1/p_1\right)\left[1-\left(1/r_c\right)\right]}$$



p-V diagram

$$p_m = \frac{\cancel{C_v}\left[(T_3 - T_2) + (C_p/C_v)(T_4 - T_3) - (T_5 - T_1)\right]}{\left[\underbrace{\cancel{C_v}(\gamma - 1)}_{=R} T_1 \Big/ p_1\right](r_c - 1)/r_c}$$

$$R = C_p - C_v \Rightarrow \frac{R}{C_v} = \frac{C_p}{C_v} - 1 = \gamma - 1$$

$$\therefore R = C_v(\gamma - 1)$$

$$\therefore p_m = \frac{p_1 r_c \left\{\left[T_2(T_3/T_2) - 1\right] + \gamma T_3\left[(T_4/T_3) - 1\right] - T_1\left[(T_5/T_1) - 1\right]\right\}}{T_1(\gamma - 1)(r_c - 1)}$$

Recalling that:

$$T_2 = T_1\, r_c^{\gamma-1} \quad ; \quad \frac{T_3}{T_2} = \alpha \quad ; \quad T_3 = T_1\, r_c^{\gamma-1}\alpha \quad ; \quad \frac{T_4}{T_3} = \rho \quad ; \quad \frac{T_5}{T_1} = \alpha\rho^{\gamma}$$

$$p_m = \frac{p_1 r_c \left\{\left[\cancel{T_1} r_c^{\gamma-1}(\alpha - 1)\right] + \gamma \cancel{T_1} r_c^{\gamma-1}\alpha\left[(\rho - 1)\right] - \cancel{T_1}\left[(\alpha\rho^{\gamma} - 1)\right]\right\}}{\cancel{T_1}(\gamma - 1)(r_c - 1)}$$

$$\therefore p_m = \frac{p_1 r_c \left\{\left[r_c^{\gamma-1}(\alpha - 1)\right] + \gamma\, r_c^{\gamma-1}\alpha(\rho - 1) - (\alpha\rho^{\gamma} - 1)\right\}}{(\gamma - 1)(r_c - 1)}$$

It is observed that:

1. For $\rho = 1$, we will get expression for p_m of Otto cycle.
2. For $\alpha = 1$, we will get expression for p_m of Diesel cycle.

1.12 Comparison of Otto, Diesel and Dual Cycles

The Otto, Diesel and dual combustion cycles may be compared based on the (i) same compression ratio and (ii) same maximum cycle temperature, keeping the heat rejection constant.

(i) Same compression ratio, with the same inlet conditions and the same heat rejection

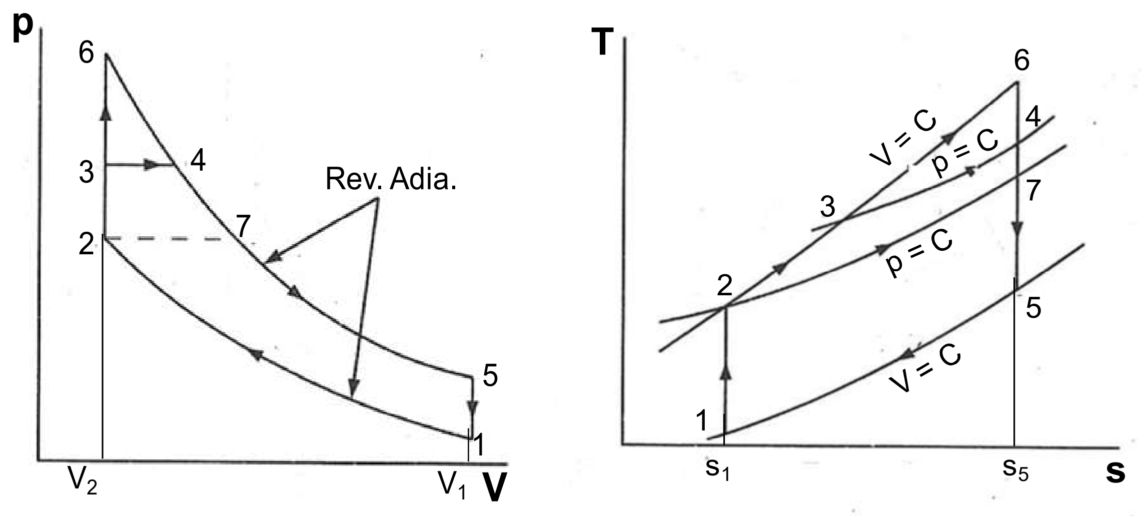


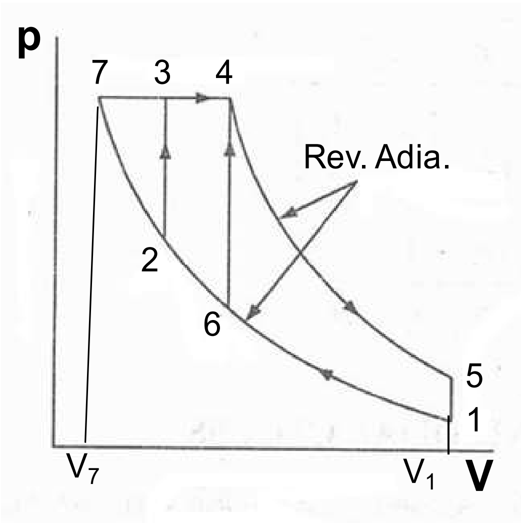
Fig. 1.10 p-V and T-s diagrams

Fig. 1.10 shows the Otto, Diesel and dual combustion cycles being plotted on p-V and T-s planes with the same compression ratio. It is observed that the area 1-2-6-5-1 which represents the net work done by the Otto cycle on p-V plane is the maximum while the area 1-2-7-5-1 which represents the work done by the Diesel cycle is the minimum. A similar observation is made with respect to the corresponding areas on T-s plane which represents the net heat transfer. Thus, it is concluded that for the same compression ratio and fixed inlet conditions and heat rejection, the Otto cycle has the maximum efficiency.

$$\therefore \eta_{Otto} > \eta_{Dual} > \eta_{Diesel}$$

(ii) Same maximum cycle pressure and temperature, with the same inlet conditions and same heat rejection

Fig. 1.11 shows the p-V and T-s planes on which the Otto, Diesel and dual combustion cycles are plotted for the same maximum cycle pressure and temperature. It is observed that the area 1-7-4-



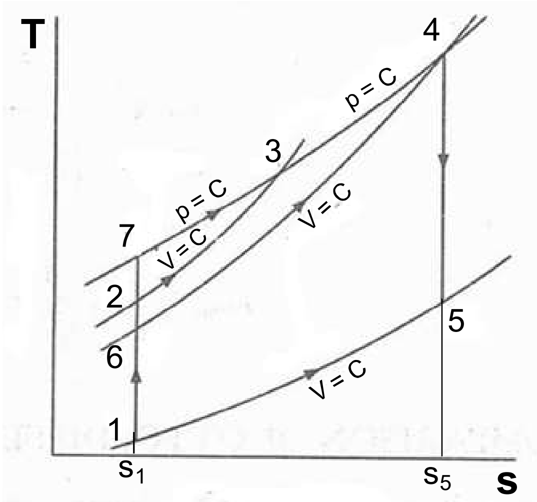


Fig. 1.11 p-V and T-s diagrams

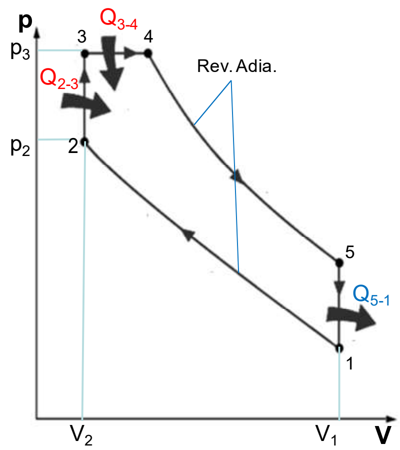
5-1 on p-V and T-s planes is the maximum while the area 1-6-4-5-1 is the minimum. This means that the net work done and net heat transferred in Diesel cycle is maximum while those of Otto cycle is minimum. Thus, it is concluded that for the same maximum cycle pressure and temperature keeping the inlet conditions and heat rejection constant, the Diesel cycle has the maximum efficiency.

$$\therefore \eta_{Diesel} > \eta_{Dual} > \eta_{Otto}$$

1.13 Numerical examples on dual combustion cycle

Numerical example 1.13.1

The compression ratio, initial pressure and temperature of an air standard dual cycle are 16, 1 bar and 50^{0}C, respectively. The maximum pressure is 70 bar. An equal amount of heat is transferred to air in both constant volume and constant pressure processes. Determine (i) the pressures and temperatures at the cardinal points of the cycle (ii) the cycle efficiency and (iii) the m.e.p of the cycle. Assume for air c_v = 0.718 kJ/kgK and c_p = 1.005 kJ/kgK.

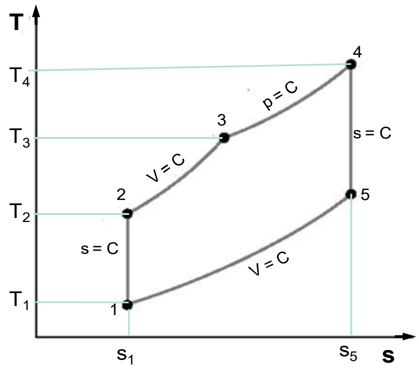


p-V diagram

Data given:

- Dual Cycle
- $r_c = (v_1 / v_2) = 16$
- p_1 = 1 bar = 1 x 10^5 N/m^2
- T_1 = 50^{0}C = 50 + 273 = 323 K
- p_3 = 70 bar = 70 x 10^5 N/m^2
- $q_{3\text{-}4} = q_{2\text{-}3} \rightarrow q_{sup} = 2 \times q_{2\text{-}3}$
- c_v = 0.718 kJ/kgK = 0.718 x 10^3 J/kgK
- c_p = 1.005 kJ/kgK = 1.005 x 10^3 J/kgK

$\rightarrow \gamma = c_p / c_v$ = 0.718 / 1.005 = 1.4



T-s diagram

To determine:

(i) Pressures and temperatures at salient points

(ii) Cycle efficiency (η)

(iii) MEP (p_m)

Solution:

Process 1-2 (Isentropic or reversible adiabatic compression)

$$\frac{p_2}{p_1} = \left(\frac{v_1}{v_2}\right)^{\gamma} = (r_c)^{\gamma} \Rightarrow p_2 = p_1 r_c^{\gamma}$$

$$\therefore p_2 = 1 \times 10^5 \times 16^{1.4} = 48.5 \times 10^5 \, N/m^2$$

and

$$\frac{T_2}{T_1} = \left(\frac{v_1}{v_2}\right)^{\gamma-1} = \left(r_c\right)^{\gamma-1} \Rightarrow T_2 = T_1 r_c^{\gamma-1}$$

$$\therefore T_2 = 323 \times 16^{1.4-1} = 979.15K$$

At State 1:

$$p_1 v_1 = RT_1 \Rightarrow v_1 = \frac{RT_1}{p_1} = \frac{0.287 \times 10^3 \times 323}{1 \times 10^5} = 0.927 m^3 / kg$$

For Constant volume process 2-3:

$$\frac{p_2}{T_2} = \frac{p_3}{T_3} \Rightarrow T_3 = T_2\left(\frac{p_3}{p_2}\right) = 979\left(\frac{70}{48.5}\right)$$

$$\therefore T_3 = 1412.99K$$

Heat Supplied, q_{2-3}:

$$q_{2-3} = c_v\left(T_3 - T_2\right) = 0.718 \times 10^3 \times \left(1412.99 - 979\right)$$

$$\therefore q_{2-3} = 311.6 \times 10^3 J / kg$$

Given that: $q_{3-4} = q_{2-3}$

$$q_{3-4} = c_p\left(T_4 - T_3\right) \Rightarrow 311.6 \times 10^3 = 1.005 \times 10^3 \times \left(T_4 - 1412.99\right)$$

$$\therefore T_4 = 1723.04K$$

For Constant pressure process 3-4:

$$\frac{v_3}{T_3} = \frac{v_4}{T_4} \Rightarrow \rho = \frac{v_4}{v_3} = \frac{T_4}{T_3} = \frac{1723.04}{1412.99} = 1.22$$

For Process 4-5 (Isentropic or reversible adiabatic expansion)

$$\frac{T_5}{T_4} = \left(\frac{v_4}{v_5}\right)^{\gamma-1} = \left(\frac{\rho}{r_c}\right)^{\gamma-1} \Rightarrow T_5 = T_4\left(\frac{\rho}{r_c}\right)^{\gamma-1}$$

$$\therefore T_5 = 1723.04 \times \left(\frac{1.22}{16}\right)^{1.4-1} = 615.54K$$

For Constant volume process 5-1:

$$\frac{p_5}{T_5} = \frac{p_1}{T_1} \Rightarrow p_5 = p_1\left(\frac{T_5}{T_1}\right) = 1\times10^5\times\left(\frac{615.54}{323}\right)$$

$$\therefore p_5 = 1.91\times10^5 N/m^2$$

Total Heat Supplied, q_{sup}:

$$q_{\sup} = 2\times q_{2-3} = 2\times311.6\times10^3 = 623.2\times10^3 J/kg$$

Heat rejected, q_{rej}:

$$q_{rej} = c_v\left(T_5 - T_1\right) = 0.718\times10^3\left(615.54-323\right)$$

$$\therefore q_{rej} = 210.04\times10^3 J/kg$$

Cycle efficiency, η:

$$\eta = 1-\frac{q_{rej}}{q_{\sup}} = 1-\frac{210.04}{623.2} = 0.663(66.3\%)$$

Net work done, w_{net}:

$$w_{net} = q_{\sup} - q_{rej} = \left(623.2-210.04\right)\times10^3 = 413.16\times10^3 J/kg$$

Stroke volume, v_s:

$$v_s = v_1 - v_2 = v_1\left[1-\left(\frac{1}{r_c}\right)\right] = 0.927\times\left[1-\left(\frac{1}{16}\right)\right]$$

$$\therefore v_s = 0.8691 m^3/kg$$

Mean effective pressure, p_m:

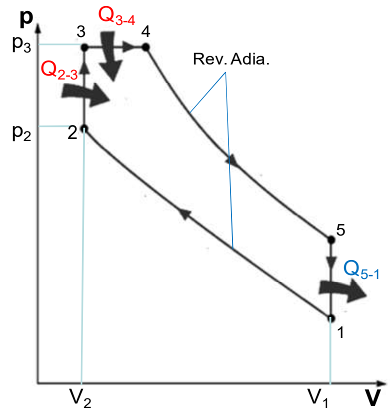
$$p_m = \frac{w_{net}}{v_s} = \frac{413.16\times10^3}{0.8691} = 4.754\times10^5 N/m^2$$

Numerical example 1.13.2

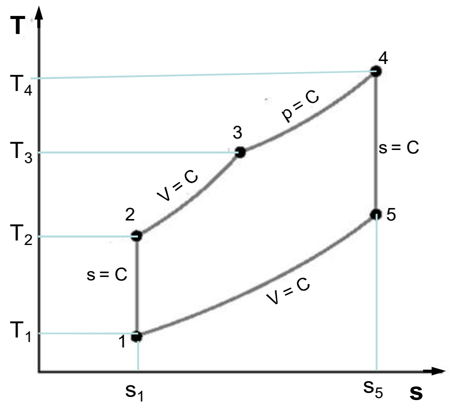
The compression ratio of a single cylinder engine operating on limited pressure cycle is 8. The maximum pressure in the cycle is limited to 55 bar. The initial pressure and temperature of air is 1 bar and 27^{0}C. Heat is added during the constant pressure process up to 3% of the stroke. Assuming the diameter and stroke of the cylinder as 25 cm and 30 cm respectively, determine (i) work done per cycle (ii) air standard efficiency and (iii) power developed, if number of working cycles are 200 per minute.

Data given:

- Limited pressure Cycle
- $r_c = (v_1 / v_2) = 8$
- $p_3 = p_4 = 55$ bar $= 55 \times 10^5$ N/m^2
- $p_1 = 1$ bar $= 1 \times 10^5$ N/m^2
- $T_1 = 27^0$C $= 27 + 273 = 300$ K
- $V_4 - V_3 = 0.03V_s$
- d = 25 cm = 25 x 10^{-2} m
- L = 30 cm = 30 x 10^{-2} m
- n = 200 cycles/min



p-V diagram



T-s diagram

To determine:

(i) Work done per cycle

(ii) Air standard efficiency

(iii) Power developed

Solution:

Stroke volume, V$_s$:

$$V_s = \frac{\pi d^2}{4} \times L = \frac{\pi \times \left(25 \times 10^{-2}\right)^2}{4} \times 30 \times 10^{-2} = 0.01473 m^3$$

Clearance volume, V_2:

$$r_c = \frac{V_1}{V_2} = \frac{V_2 + V_s}{V_2} = 1 + \frac{V_s}{V_2} \Rightarrow V_2 = \frac{V_s}{r_c - 1} = \frac{0.01473}{8-1}$$
$$\therefore V_2 = 2.104 \times 10^{-3} m^3$$

Cut-off ratio, ρ:

$$or\ \frac{V_4}{V_3} - 1 = 0.03\frac{V_s}{V_3} \Rightarrow \rho - 1 = 0.03\frac{V_s}{V_2} \Rightarrow \rho = 0.03\frac{V_s}{V_2} + 1$$
$$\therefore \rho = 0.03 \times \frac{0.01473}{2.104 \times 10^{-3}} + 1 = 1.21$$

Total volume, V_1:

$$or\ \frac{V_1}{V_2} = 8 \Rightarrow V_1 = 8V_2 = 8 \times 2.104 \times 10^{-3} = 0.01683 m^3$$

For process 1-2 (Isentropic or reversible adiabatic compression)

$$\frac{p_2}{p_1} = \left(\frac{V_1}{V_2}\right)^\gamma = r_c^\gamma \Rightarrow p_2 = p_1 r_c^\gamma = 1 \times 10^5 \times 8^{1.4}$$
$$\therefore p_2 = 18.38 \times 10^5 N/m^2$$

and

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma-1} = r_c^{\gamma-1} \Rightarrow T_2 = T_1 r_c^{\gamma-1} = 300 \times 8^{1.4-1}$$
$$\therefore T_2 = 689.22 K$$

Explosion ratio, α:

$$\alpha = \frac{p_3}{p_2} = \frac{55}{18.38} = 2.99$$

Air standard efficiency, η:

$$\eta = 1 - \frac{1}{r_c^{\gamma-1}}\left[\frac{\alpha\rho^\gamma - 1}{(\alpha - 1) + \gamma\alpha(\rho - 1)}\right]$$

$$\eta = 1 - \frac{1}{8^{1.4-1}}\left[\frac{2.99\times1.21^{1.4}-1}{(2.99-1)+1.4\times2.99\times(1.21-1)}\right]$$
$$\therefore \eta = 0.5593(55.93\%)$$

For process 2-3 (Isochoric or constant volume heat supply)

$$\frac{p_2}{T_2} = \frac{p_3}{T_3} \Rightarrow T_3 = T_2\left(\frac{p_3}{p_2}\right) = T_2\alpha \Rightarrow T_3 = 689.22\times2.99$$
$$\therefore T_3 = 2060.76K$$

For process 3-4 (Isobaric or constant pressure heat supply)

$$\frac{V_3}{T_3} = \frac{V_4}{T_4} \Rightarrow T_4 = T_3\left(\frac{V_4}{V_3}\right) = T_3\rho \Rightarrow T_4 = 2060.76\times1.21$$
$$\therefore T_4 = 2493.52K$$

Heat supplied, Q_{sup}:

$$Q_{sup} = Q_{2-3} + Q_{3-4} = c_V\left(T_3 - T_2\right) + c_P\left(T_4 - T_3\right)$$
$$= 0.718\times10^3\times(2060.76-689.22)+1.005\times10^3\times(2493.52-2060.76)$$
$$\therefore Q_{sup} = 984.77 + 434.92 = 1419.69\times10^3 J$$

Net work done, W:

$$\eta = \frac{W_{net}}{Q_{sup}} \Rightarrow W_{net} = \eta\times Q_{sup} = 0.5593\times1419.69\times10^3$$
$$\therefore W_{net} = 794.03\times10^3 J/cycle$$

Power developed, P:

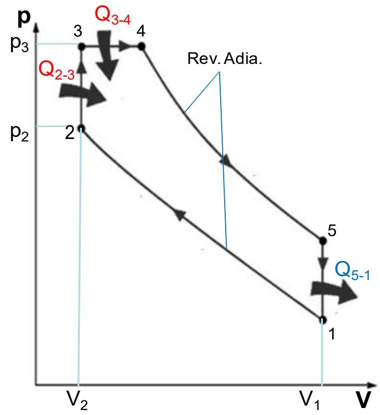
$$P = \frac{W_{net}\times n}{60} = \frac{794.03\times10^3\times200}{60}$$
$$\therefore P = 2646.77\times10^3 J/s(W)$$

Numerical example 1.13.3

A diesel engine working on semi-diesel cycle has a swept volume of 0.0055 m^3 and clearance volume of 0.00035 m^3. The maximum cycle pressure is 60 bar and fuel injection ends at 6% of the stroke. The pressure and temperature at the beginning of the compression are 75^0C and 0.95 bar. Determine the air standard efficiency and mean effective pressure of the cycle.

Data given:

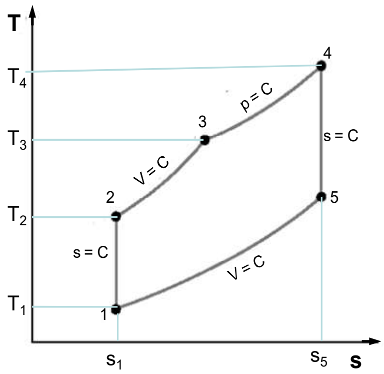
- Semi-diesel Cycle
- $V_s = 0.0055$ m^3
- $V_{c=3=2} = 0.00035$ m^3
- $p_3 = p_4 = 60$ bar $= 60 \times 10^5$ N/m^2
- $V_4 - V_{3=2} = 0.06V_s$
- $p_1 = 0.95$ bar $= 0.95 \times 10^5$ N/m^2
- $T_1 = 75^0C = 75 + 273 = 348$ K



p-V diagram

For air, assume that:

- $c_p = 1.005$ kJ/kgK $= 1.005 \times 10^3$ J/kgK
- $c_v = 0.718$ kJ/kgK $= 0.718 \times 10^3$ J/kgK
- $R = 0.287$ kJ/kgK $= 0.287 \times 10^3$ J/kgK
- $\gamma = 1.4$



T-s diagram

To determine:

(i) Air standard efficiency

(ii) Mean effective pressure

Solution:

Compression ratio, r_c:

$$r_c = 1 + \frac{V_s}{V_2} = 1 + \frac{0.0055}{0.00035} = 16.71$$

Cut-off ratio, ρ:

$$\rho = \frac{V_4}{V_{3=2}} = \frac{0.06V_s + V_{3=2}}{V_{3=2}} = \frac{(0.06 \times 0.0055) + 0.00035}{0.00035}$$
$$\therefore \rho = 1.94$$

For process 1-2 (Isentropic or reversible adiabatic compression)

$$\frac{p_2}{p_1} = \left(\frac{V_1}{V_2}\right)^{\gamma} = r_c^{\gamma} \Rightarrow p_2 = p_1 r_c^{\gamma} = 0.95 \times 10^5 \times 16.71^{1.4}$$
$$\therefore p_2 = 48.97 \times 10^5 \, N/m^2$$

Explosion ratio, α:

$$\alpha = \frac{p_3}{p_2} = \frac{60}{48.97} = 1.23$$

Air standard efficiency, η:

$$\eta = 1 - \frac{1}{r_c^{\gamma-1}}\left[\frac{\alpha\rho^{\gamma} - 1}{(\alpha - 1) + \gamma\alpha(\rho - 1)}\right]$$

$$\eta = 1 - \frac{1}{16.71^{1.4-1}}\left[\frac{1.23 \times 1.94^{1.4} - 1}{(1.23 - 1) + 1.4 \times 1.23 \times (1.94 - 1)}\right]$$
$$\therefore \eta = 0.6298(62.98\%)$$

Mean effective pressure, p_m:

$$p_m = \frac{p_1 r_c \left\{\left[r_c^{\gamma-1}(\alpha - 1)\right] + \gamma\, r_c^{\gamma-1}\alpha(\rho - 1) - (\alpha\rho^{\gamma} - 1)\right\}}{(\gamma - 1)(r_c - 1)}$$

$$p_m = \frac{0.95 \times 10^5 \times 16.71\left\{\left[16.71^{1.4-1}(1.23 - 1)\right] + 1.4 \times 16.71^{1.4-1} \times 1.23 \times (1.94 - 1) - (1.23 \times 1.94^{1.4} - 1)\right\}}{(1.4 - 1)(16.71 - 1)}$$

$$\therefore p_m = 9.072 \times 10^5 \, N/m^2$$

1.14 Gas turbine cycles

- Turbine is a machine which extracts power from flowing fluids.
- In a gas turbine, gases at high-pressure and high-temperature are expanded through several stages to produce power.
- Gas turbine power plants are used for aviation applications and power generation.

1.14.1 Advantages of gas turbines

- Higher power-to-weight ratio compared to reciprocating engines
- Smaller size for the same power produced.
- Lower emission levels.
- High rotational speed.

1.14.2 Disadvantages of gas turbines

- More expensive.
- Consumes more fuel during idling.
- Works best at constant load than fluctuating loads.

1.15 Open cycle gas turbine power plant (Joule cycle)

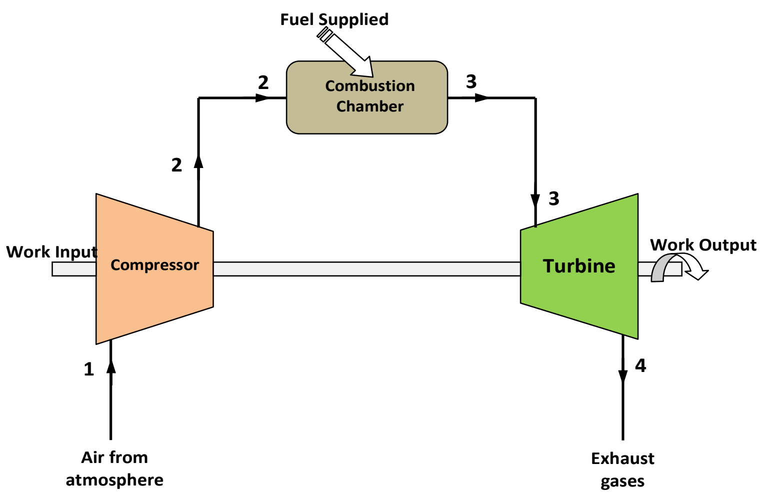


Fig. 1.12 Schematic diagram of open cycle gas turbine power plant

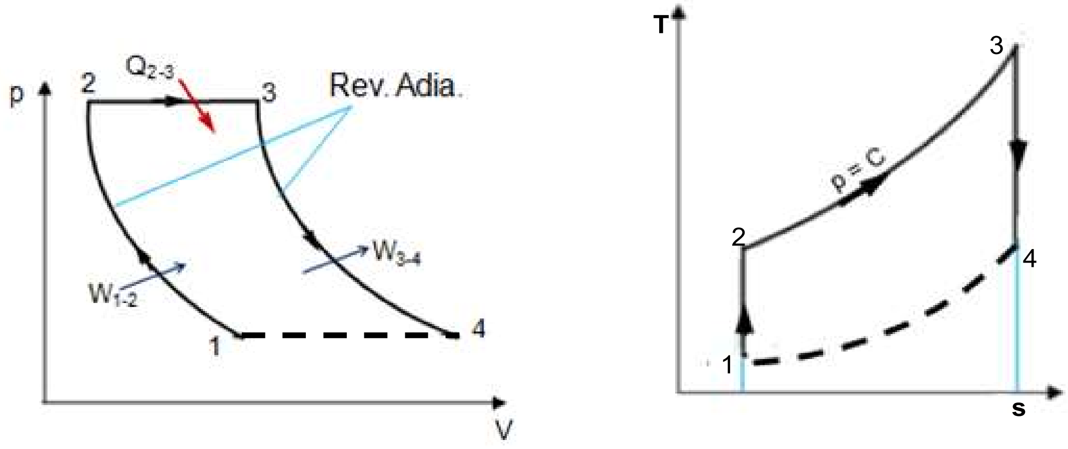


Fig. 1.13 p-V and T-s diagrams of open cycle gas turbine power plant

The open cycle gas turbine power plant, also referred to as Joule cycle, is shown schematically in Fig. 1.12. The p-V and T-s diagrams are shown in Fig. 1.13. The working of open cycle gas turbine power plant is explained as follows:

- **Process 1-2: Isentropic or reversible adiabatic compression**
  Air is taken from the atmosphere by the compressor and compresses isentropically to pressure p_2 and temperature T_2.

- **Process 2-3: Constant pressure heat supply**
  The high pressure and temperature air enters the combustion chamber wherein the fuel is supplied. The fuel is burnt in the presence of hot air and the combustion gases are produced. The pressure and temperature of the combustion gases increases to p_3 and T_3 respectively.

- **Process 3-4: Isentropic or reversible adiabatic expansion**
  The high-pressure and high-temperature combustion gases enter a turbine where it expands to a low-pressure (equal to or a little above the atmospheric pressure) gas.

- **Process 4-1:**
  The exhaust gases are released to the surroundings. The dashed line 4-1 represents that the cycle is open.

1.16 Closed cycle gas turbine power plant (Brayton cycle)

- Closed Cycle is the air-standard cycle of an actual gas turbine plant where the combustion and exhaust of gases are replaced by constant pressure heat addition and heat rejection processes.
- The ideal cycle in which the working fluid (air) undergoes thermodynamic processes in a closed loop to produce the net power is the Brayton Cycle.

1.16.1 Advantages of closed cycle gas turbine power plant

1. The use of higher pressure throughout the cycle reduces the plant size for a given output.
2. It avoids corrosion and erosion of turbine blades due to contaminated gases as indirect heating is used in the cycle.
3. No requirement of filtration of incoming air.
4. Cheap solid fuels like coal, wood can be economically used.
5. Low maintenance cost and high reliability.

1.16.2 Disadvantages of closed cycle gas turbine power plant

1. This cycle is not suitable for air craft applications as huge quantity of cooling water is required in the cooler.
2. The weight of the plant per unit of power developed is high.
3. It uses a large air-heater compared to combustion chamber of open cycle, because heat transfer coefficient in the heat exchanger is low.
4. The installation cost is high.

1.17 Ideal Brayton cycle (Closed cycle)

An ideal Brayton cycle consists of two reversible adiabatic (isentropic) processes and two constant pressure (isobaric) processes. These thermodynamic processes are executed in steady flow devices in cyclic order.

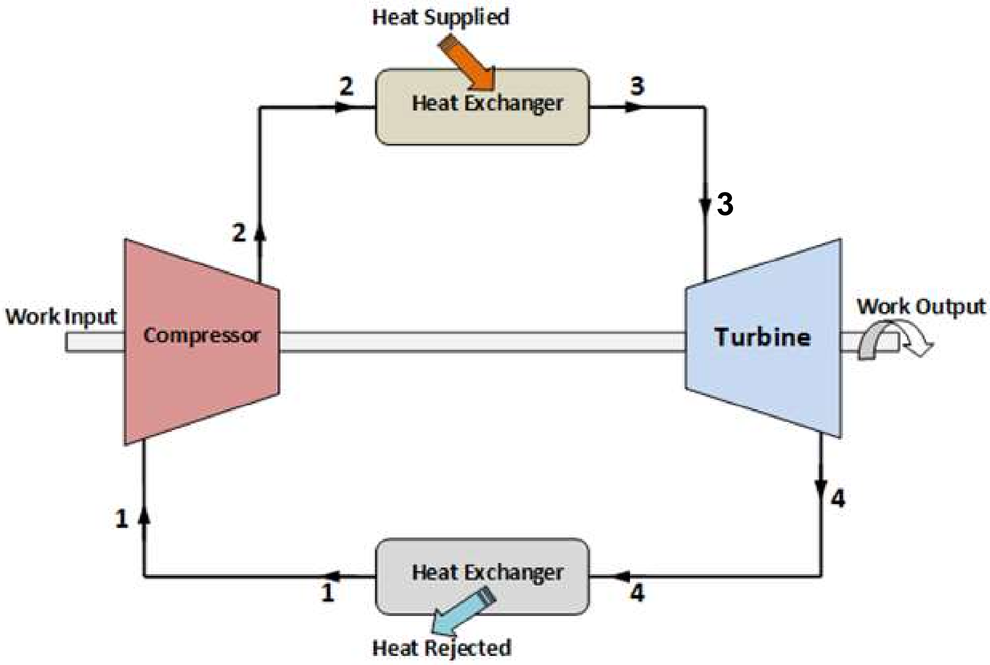


Fig. 1.14 Schematic diagram of closed cycle gas turbine power plant

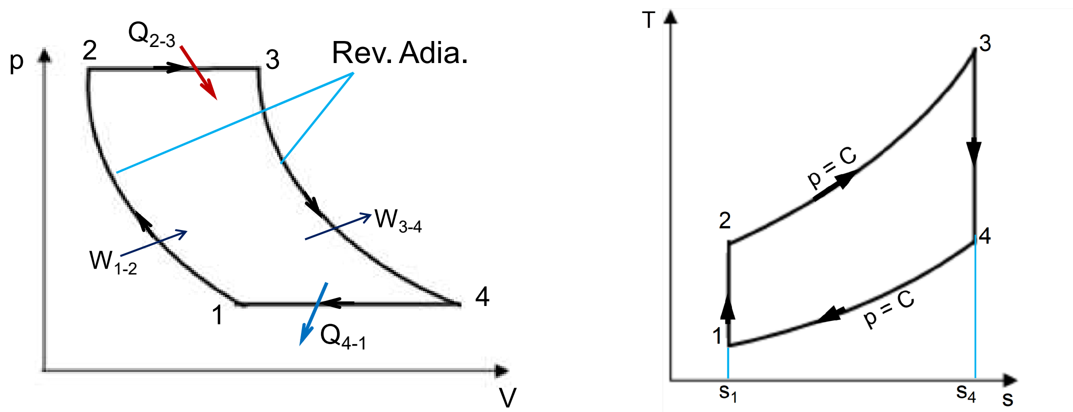


Fig. 1.15 p-V and T-s diagrams of closed cycle gas turbine power plant

The closed cycle gas turbine power plant, also referred to as Brayton cycle, is schematically shown in Fig. 1.14. The thermodynamic processes comprising the Brayton cycle are plotted on p-V and T-s planes as shown in Fig. 1.15. The working of the cycle is explained as follows:

Process 1-2: Isentropic or reversible adiabatic compression in a compressor

The working fluid (air) enters the compressor at state 1 and is compressed isentropically (Rev. Adiabatic) to state 2.

Process 2-3: Isobaric or constant pressure heat supply

The high-pressure air then enters the heat exchanger where the heat is added to the air from an external source.

Process 3-4: Isentropic or reversible adiabatic expansion in a Turbine

The high-pressure and high-temperature air at state 3 is expanded isentropically (Rev. Adiabatic) to state 4 by a turbine producing the work.

Process 4-1: Isobaric or constant pressure heat rejection

The low-pressure and low-temperature air enters the heat exchanger at state 4 and the heat is rejected to an external sink. The air leaves the heat exchanger at state 1 and re-enters the compressor, thus completing the cycle.

1.17.1 Thermodynamic analysis of an ideal Brayton cycle

Heat Supplied: $Q_{\sup} = Q_{2-3} = mc_p\left(T_3 - T_2\right)$

Heat Rejected: $Q_{rej} = Q_{4-1} = mc_p\left(T_4 - T_1\right)$

Net Work Done: $W_{net} = Q_{\sup} - Q_{rej} = mc_p\left[\left(T_3 - T_2\right) - \left(T_4 - T_1\right)\right]$

Cycle Efficiency:

$$\eta = \frac{W_{net}}{Q_{\sup}} = 1 - \frac{Q_{rej}}{Q_{\sup}} = 1 - \frac{\cancel{mc}_p\left(T_4 - T_1\right)}{\cancel{mc}_p\left(T_3 - T_2\right)} = 1 - \frac{\left(T_4 - T_1\right)}{\left(T_3 - T_2\right)}$$

$$\eta = 1 - \frac{T_1\left(T_4/T_1 - 1\right)}{T_2\left(T_3/T_2 - 1\right)} \qquad \text{------ (7)}$$

Processes 1-2 and 3-4 are isentropic:

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} \quad \text{and} \quad \frac{T_3}{T_4} = \left(\frac{p_3}{p_4}\right)^{\frac{\gamma-1}{\gamma}}$$

Processes 2-3 and 4-1 are isobaric:

$$p_2 = p_3 \quad \text{and} \quad p_4 = p_1$$

Therefore:

$$\frac{T_2}{T_1} = \frac{T_3}{T_4} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} = \left(r_p\right)^{\frac{\gamma-1}{\gamma}}$$

$$\Rightarrow \frac{T_4}{T_1} = \frac{T_3}{T_2}$$

Substituting in Eqn. (7):

$$\eta = 1 - \frac{T_1}{T_2} = 1 - \frac{1}{T_2/T_1}$$

$$\therefore \eta = 1 - \frac{1}{\left(r_p\right)^{\frac{\gamma-1}{\gamma}}}$$

Observe that the thermal efficiency of an ideal Brayton gas turbine cycle increases with the pressure ratio.

1.17.2 Effect of pressure ratio on work output of Brayton cycle

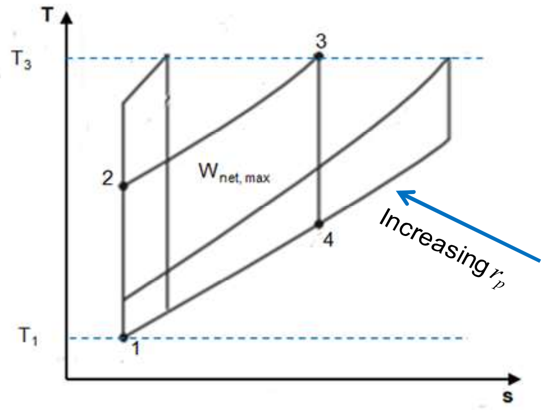


Fig. 1.16 T-s diagram showing the effect of pressure ratio

- T-s plot of Brayton cycle working between fixed temperature limits and increasing pressure ratio is shown in Fig. 1.16.
- It is observed that the net work output per cycle increases with the pressure ratio, reaches a maximum, and then starts to decrease.
- Area 1-2-3-4 gives the maximum work output.

1.17.3 Optimum pressure ratio for maximum work output

The net work done by the cycle is given by

$$W_{net} = W_t - W_c = mc_p\left[(T_3 - T_4) - (T_2 - T_1)\right] \qquad \text{------ (8)}$$

$$\therefore W_{net} = mc_p\left[T_3\left(1 - \frac{T_4}{T_3}\right) - T_1\left(\frac{T_2}{T_1} - 1\right)\right]$$

From processes 1-2 and 3-4 (Isentropic or reversible adiabatic):

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} = \left(r_p\right)^{\frac{\gamma-1}{\gamma}} \qquad \text{------ (9)}$$

and

$$\frac{T_4}{T_3} = \left(\frac{p_4}{p_3}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{p_1}{p_2}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{1}{r_p}\right)^{\frac{\gamma-1}{\gamma}} \qquad \text{------ (10)}$$

Substituting Eqns . (9) and (10) into (8):

$$W_{net} = mc_p\left[T_3\left(1-\frac{1}{r_p^{\frac{\gamma-1}{\gamma}}}\right)-T_1\left(r_p^{\frac{\gamma-1}{\gamma}}-1\right)\right]$$

Let: $x=\dfrac{\gamma-1}{\gamma}$ so that

$$W_{net} = mc_p\left[T_3\left(1-r_p^{-x}\right)-T_1\left(r_p^{x}-1\right)\right] \qquad \text{------ (11)}$$

Optimum pressure ratio is determined by differentiating the Eqn. (11) with r_p and then equating it to zero.

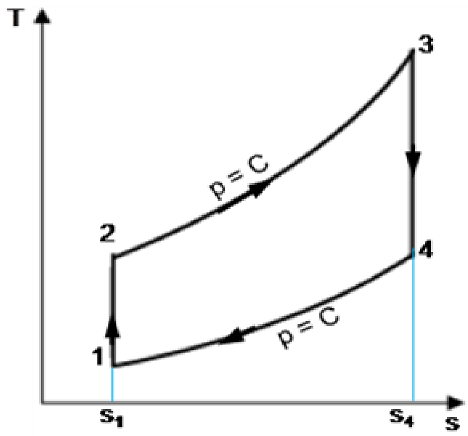
$$\frac{dW_{net}}{dr_p}=\frac{d\left\{mc_p\left[T_3\left(1-r_p^{-x}\right)-T_1\left(r_p^{x}-1\right)\right]\right\}}{dr_p}=0$$

$$T_3\left[-\left(-x\cdot r_p^{-x-1}\right)\right]-T_1\left[\left(x\cdot r_p^{x-1}\right)\right]=0$$

$$\frac{T_3}{T_1}=\frac{\not{x}\cdot r_p^{x-1}}{\not{x}\cdot r_p^{-x-1}}=r_p^{x-1+x+1}=r_p^{2x}$$

$$r_{p_{opt}}=\left(\frac{T_3}{T_1}\right)^{\frac{1}{2x}}=\left(\frac{T_3}{T_1}\right)^{\frac{1}{2\left(\frac{\gamma-1}{\gamma}\right)}}$$

$$\therefore r_{p_{opt}}=\left(\frac{T_3}{T_1}\right)^{\frac{\gamma}{2(\gamma-1)}}$$



T-s diagram

1.17.4 Deviation of actual cycle from an ideal Brayton cycle

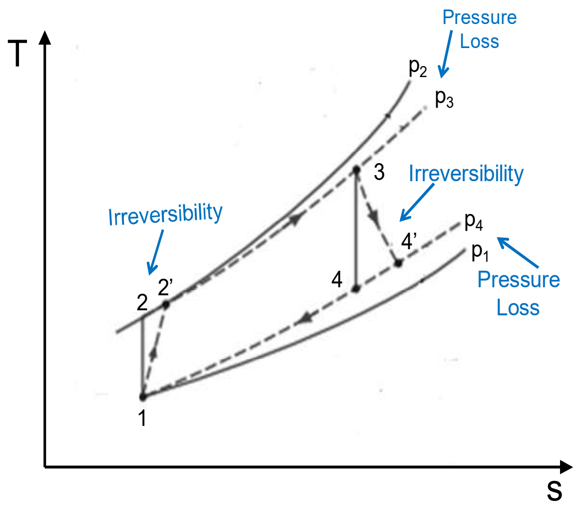


Fig. 1.17 T-s diagram showing the deviation of actual cycle

1. The compression and expansion processes are not isentropic. There is an increase in entropy at the end of each process.
2. Due to this, the actual work input to the compressor is more, and the actual work output from the turbine is less. The actual compression and expansion processes are shown by 1-2' and 3-4' in Fig. 1.17.
3. This deviation is accounted by defining the 'isentropic efficiency' of compressor and turbine.
4. There is a small pressure drop in the heat addition (from p_2 to p_3) and heat rejection (from p_4 to p_1) processes due to frictional effects.

The isentropic efficiencies of compressor and turbine are given by:

$$\eta_c = \frac{w}{w'} = \frac{c_p\left(T_2 - T_1\right)}{c_p\left(T_{2'} - T_1\right)} = \frac{h_2 - h_1}{h_{2'} - h_1}$$

$$\eta_t = \frac{w'}{w} = \frac{c_p\left(T_3 - T_{4'}\right)}{c_p\left(T_3 - T_4\right)} = \frac{h_3 - h_{4'}}{h_3 - h_4}$$

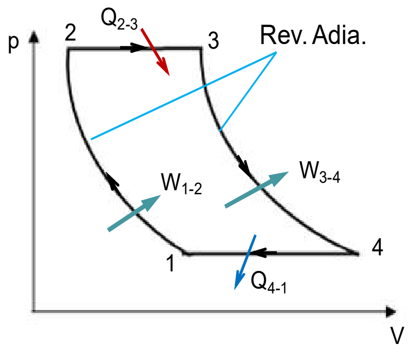
1.18 Numerical examples on Brayton cycle

Numerical example 1.18.1

Air enters the compressor of an ideal air standard Brayton cycle at 100 kPa, 300 K with a volumetric flow rate of 6 m^3/s. The compressor pressure ratio is 10. The turbine inlet temperature is 1500 K. Determine (i) thermal efficiency (ii) work ratio (iii) power developed.

Data given:

- Brayton Cycle
- $p_1 = 100 \times 10^3$ N/m^2
- $T_1 = 300$ K
- $V_1 = 6$ m^3/s
- $r_p = \frac{p_2}{p_1} = \frac{p_3}{p_4} = 10$
- $T_3 = 1500$ K



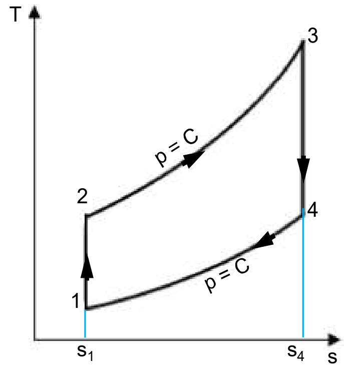
p-V diagram

To determine:

(i) Thermal efficiency

(ii) Work ratio

(iii) Power developed



T-s diagram

Solution:

Process 1-2 (Isentropic Compression):

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} \Rightarrow T_2 = T_1 r_p^{\frac{\gamma-1}{\gamma}} \Rightarrow T_2 = 300 \times (10)^{\frac{1.4-1}{1.4}} = 579.19K$$

Compressor Work, w_c:

$$w_c = c_p (T_2 - T_1) = 1.005 \times 10^3 \times (579.19 - 300)$$

$$\therefore w_c = 280.59 \times 10^3 \, J/kg$$

Process 3-4 (Rev. Adia. Expn.):

$$\frac{T_4}{T_3} = \left(\frac{p_4}{p_3}\right)^{\frac{\gamma-1}{\gamma}} = \frac{1}{r_p^{\frac{\gamma-1}{\gamma}}} \Rightarrow T_4 = \frac{T_3}{r_p^{\frac{\gamma-1}{\gamma}}} = \frac{1500}{10^{\frac{1.4-1}{1.4}}} = 776.95K$$

Turbine Work, w_t:

$$w_t = c_p\left(T_3 - T_4\right) = 1.005\times10^3 \times\left(1500 - 776.95\right)$$

$$w_t = 726.67\times10^3 J/kg$$

Net Work done w_{net}:

$$w_{net} = w_t - w_c = \left(726.67 - 280.59\right)\times10^3$$

$$\therefore w_{net} = 446.08\times10^3 J/kg$$

Heat Supplied, q_{sup}:

$$Q_{2-3} = c_p\left(T_3 - T_2\right) = 1.005\times10^3 \times\left(1500 - 579.19\right)$$

$$\therefore Q_{2-3} = 920.81\times10^3 J/kg$$

Thermal Efficiency, η:

$$\eta = \frac{w_{net}}{q_{\sup}} = \frac{446.08}{920.81} = 0.4844(48.44\%)$$

Work Ratio, WR:

$$WR = \frac{w_{net}}{w_t} = \frac{446.08}{726.67}$$

$$\therefore WR = 0.6139$$

Mass Flow Rate of Air, m:

$$\dot{m} = \frac{p_1 V_1}{RT_1} = \frac{100\times10^3\times6}{0.287\times10^3\times300}$$

$$\therefore \dot{m} = 6.97 kg/s$$

Power Developed, P:

$$P = \dot{m}\, w_{net} = 6.97\times446.08\times10^3$$

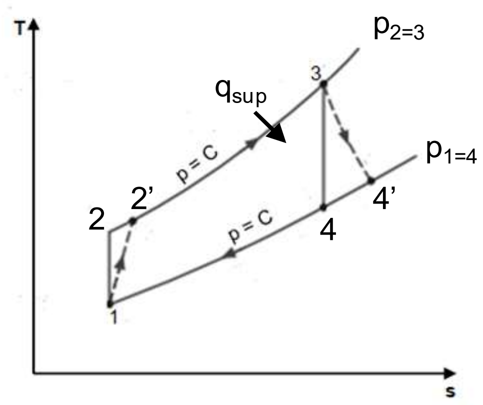
$$\therefore P = 3109.18\times10^3 J/s$$

Numerical example 1.18.2

Air enters the compressor of an open cycle constant pressure gas turbine at a pressure of 1 bar and temperature of 20^0C. The pressure of air after compression is 4 bar. The isentropic efficiencies of compressor and turbine are 80% and 85% respectively. The air to fuel ratio is 90:1. Calorific value of the fuel is 42,000 kJ/kg. If the flow rate of air is 3 kg/s, find (i) power developed (ii) thermal efficiency of the cycle.

Data given:

- Brayton Cycle
- $p_1 = 1 \times 10^5$ N/m^2
- $T_1 = 20 + 273 = 293$ K
- $p_2 = 4 \times 10^5$ N/m^2
- $\eta_c = 0.80$
- $\eta_t = 0.85$
- $m_a / m_f = 90$
- CV = 42,000 x 10^3 J/kg
- $\dot{m}_a = 3$ kg/s



T-s diagram

For air, assume that:

- $c_p = 1.005 \times 10^3$ J/kgK
- $\gamma = 1.4$

To determine:

(i) Power developed

(ii) Thermal efficiency

Solution:

For process 1-2 (Isentropic or reversible adiabatic compression)

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} = r_p^{\frac{\gamma-1}{\gamma}} \Rightarrow T_2 = T_1 r_p^{\frac{\gamma-1}{\gamma}}$$

$$\therefore T_2 = 293 \times (4)^{\frac{1.4-1}{1.4}} = 435.39K$$

Actual temperature at compressor outlet, $T_{2'}$:

We know that compressor efficiency, η_c is given by:

$$\eta_c = \frac{T_2 - T_1}{T_{2'} - T_1} \Rightarrow 0.8 = \frac{435.39 - 293}{T_{2'} - 293}$$

$$\therefore T_{2'} = 470.99K$$

Compressor Work, w_c:

$$w_c = c_p\left(T_{2'} - T_1\right) = 1.005 \times 10^3 \times \left(470.99 - 293\right)$$

$$\therefore w_c = 178.88 \times 10^3 J/kg$$

Turbine inlet temperature, T_3:

We know that

Heat Supplied (by fuel) = Heat of Combustion Gases

$$\dot{m}_f \times CV = \left(\dot{m}_f + \dot{m}_a\right) c_p\left(T_3 - T_{2'}\right)$$

$$CV = \left(1 + \frac{\dot{m}_a}{\dot{m}_f}\right) c_p\left(T_3 - T_{2'}\right)$$

$$\Rightarrow 42,000 \times 10^3 = \left(1 + 90\right) \times 1.005 \times 10^3 \times \left(T_3 - 470.99\right)$$

$$\therefore T_3 = 930.23K$$

For process 3-4 (Isentropic or reversible adiabatic expansion)

$$\frac{T_4}{T_3} = \left(\frac{p_4}{p_3}\right)^{\frac{\gamma-1}{\gamma}} = \frac{1}{r_p^{\frac{\gamma-1}{\gamma}}} \Rightarrow T_4 = \frac{T_3}{r_p^{\frac{\gamma-1}{\gamma}}} = \frac{930.23}{4^{\frac{1.4-1}{1.4}}} = 626.01K$$

Actual temperature at turbine outlet, $T_{4'}$:

We know that Turbine Efficiency, η_t is given by:

$$\eta_t = \frac{T_3 - T_{4'}}{T_3 - T_4} \Rightarrow 0.85 = \frac{930.23 - T_{4'}}{930.23 - 626.01}$$

$$\therefore T_{4'} = 671.64K$$

Turbine Work, w_t:

$$w_t = c_p\left(T_3 - T_{4'}\right) = 1.005\times10^3\times\left(930.23 - 671.64\right)$$

$$\therefore w_t = 259.88\times10^3 J/kg$$

Net Work Done, w_{net}:

$$w_{net} = w_t - w_c = \left(259.88 - 178.88\right)\times10^3$$

$$\therefore w_{net} = 81\times10^3 J/kg$$

Heat Supplied, q_{sup}:

$$q_{sup} = q_{2'-3} = c_p\left(T_3 - T_{2'}\right) = 1.005\times10^3\times\left(930.23 - 470.99\right)$$

$$\therefore q_{sup} = 461.54\times10^3 J/kg$$

Thermal Efficiency, η:

$$\eta = \frac{w_{net}}{q_{sup}} = \frac{81}{461.54} = 0.1755(17.55\%)$$

Power Developed, P:

$$P = \dot{m}_a\, w_{net} = 3\times81\times10^3 = 243\times10^3 J/s$$

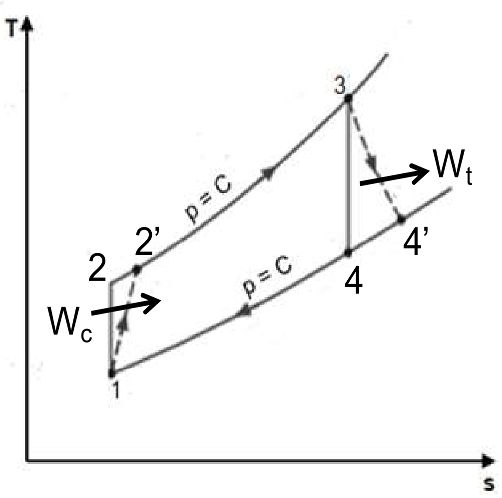
Numerical example 1.18.3

In a simple gas turbine unit, the isentropic discharge temperature of air flowing out of the compressor is 195^0C, while the discharge temperature is 240^0C. Air conditions at the compressor inlet are 1 bar and 17^0C. If the air fuel ratio is 75 and net power output from the unit is 650 kW, compute (i) isentropic efficiencies of the compressor and turbine (ii) overall cycle efficiency. Calorific value of the fuel used is 46110 kJ/kg and the unit consumes 312 kg/h of fuel. Assume for gases, c_p = 1.09 kJ/kgK and γ = 1.32, and for air c_p = 1.005 kJ/kgK and γ = 1.4.

Data given:

- Brayton Cycle
- $T_2 = 195 + 273 = 468$ K
- $T_{2'} = 240 + 273 = 513$ K

- $p_1 = 1 \times 10^5$ N/m^2
- $T_1 = 17 + 273 = 290$ K
- $\dot{m}_a / \dot{m}_f = 75$
- P = 350 x 10^3 W
- CV = 46,110 x 10^3 J/kg
- $\dot{m}_f$ = 312/3600 = 0.087 kg/s
- $\dot{m}_a$ = 75 x 0.087 = 6.525 kg/s



T-s diagram

Given assumptions:

For gases:

- c_p = 1.09 kJ/kgK = 1.09 x 10^3 J/kgK
- $\gamma = 1.32$

For air:

- c_p = 1.005 kJ/kgK = 1.005 x 10^3 J/kgK
- $\gamma = 1.4$

To determine:

(i) Isentropic efficiencies of compressor and turbine

(ii) Overall cycle efficiency

Solution:

Pressure ratio, r_p:

$$r_p = \frac{p_2}{p_1} = \left(\frac{T_2}{T_1}\right)^{\left(\frac{\gamma}{\gamma-1}\right)_{air}} = \left(\frac{468}{290}\right)^{\left(\frac{1.4}{1.4-1}\right)_{air}}$$

$$\therefore r_p = 5.34$$

Compressor efficiency, η_c:

$$\eta_c = \frac{T_2 - T_1}{T_{2'} - T_1} = \frac{468 - 290}{513 - 290}$$

$$\therefore \eta_c = 0.7982 (79.82\%)$$

Compressor work, W_c:

$$W_c = \dot{m}_a \, c_{p_{air}} \left(T_{2'} - T_1\right)$$
$$= 6.525 \times 1.005 \times 10^3 \times \left(513 - 290\right)$$
$$\therefore W_c = 1462.35 \times 10^3 W$$

Turbine inlet temperature, T_3:

We know that:

Heat Supplied (by fuel) = Heat of Combustion Gases

$$\dot{m}_f \times CV = \left(\dot{m}_f + \dot{m}_a\right) c_{p_{gases}} \left(T_3 - T_{2'}\right)$$
$$0.087 \times 46110 \times 10^3 = \left(0.087 + 6.525\right) \times 1.09 \times 10^3 \times \left(T_3 - 513\right)$$
$$\therefore T_3 = 1069.62K$$

Turbine work, W_t:

$$W_t = W_{net} + W_c = \left(650 + 1462.35\right) \times 10^3$$
$$\therefore W_t = 2112.35 \times 10^3 W$$

Actual turbine outlet temperature, $T_{4'}$:

$$W_t = \dot{m}_g \, c_{p_{gases}} \left(T_3 - T_{4'}\right) \qquad where \;\; \dot{m}_g = \left(\dot{m}_a + \dot{m}_f\right)$$
$$2112.35 \times 10^3 = \left(6.525 + 0.087\right) \times 1.09 \times 10^3 \times \left(1069.62 - T_{4'}\right)$$
$$\therefore T_{4'} = 776.53K$$

For process 3-4 (Isentropic or reversible adiabatic expansion)

$$\frac{T_4}{T_3} = \left(\frac{p_4}{p_3}\right)^{\left(\frac{\gamma-1}{\gamma}\right)_{gases}} = \frac{1}{r_p^{\left(\frac{\gamma-1}{\gamma}\right)_{gases}}} \Rightarrow T_4 = \frac{T_3}{r_p^{\left(\frac{\gamma-1}{\gamma}\right)_{gases}}} = \frac{1069.62}{5.34^{\frac{1.32-1}{1.32}}} = 712.65K$$

Turbine efficiency, η_t:

$$\eta_t = \frac{T_3 - T_{4'}}{T_3 - T_4} = \frac{1069.62 - 776.53}{1069.62 - 712.65}$$
$$\therefore \eta_t = 0.8210(82.1\%)$$

Heat supplied, Q_{sup}:

$$Q_{sup} = \dot{m}_f \times CV = 0.087 \times 46110 \times 10^3$$
$$\therefore Q_{sup} = 4011.57 \times 10^3 \, J/s$$

Cycle efficiency, η:

$$\eta = \frac{W_{net}}{Q_{sup}} = \frac{650 \times 10^3}{4011.57 \times 10^3}$$
$$\therefore \eta = 0.1620 (16.20\%)$$

Numerical example 1.18.4

Air is drawn in a gas turbine at 18^0C and 1 bar and leaves the compressor at 5 bar. Data observed are: Temperature of gases entering the turbine = 678^0C, Pressure loss in combustion chamber = 0.1 bar, Efficiency of compressor = 85%, Efficiency of combustion = 85%, Efficiency of turbine = 80%, γ = 1.4 for air, c_p = 1.024 kJ/kgK, γ = 1.3 for gases. Determine (i) quantity of air if plant develops 1065 kW (ii) Heat supplied per kg of air circulate (iii) Thermal efficiency of the cycle.

Data given:

- Brayton Cycle
- T_1 = 18 +273 = 291 K
- $p_1 = 1 \times 10^5$ N/m^2
- $p_2 = 5 \times 10^5$ N/m^2
- $T_{3'}$ = 678 +273 = 951 K
- Pressure Loss = 0.1×10^5 N/m^2
- η_c = 0.85
- η_{comb} = 0.85
- η_t = 0.8
- P = 1065×10^3 W

Given:

For gases:

- $c_p = 1.024$ kJ/kgK = 1.024 x 10^3 J/kgK
- $\gamma = 1.3$

For air:

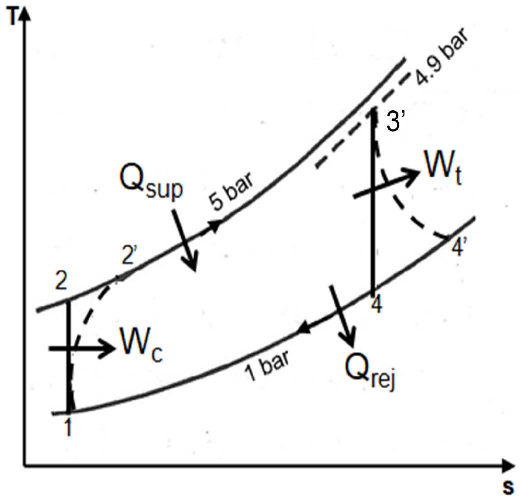
- $\gamma = 1.4$

To determine:

(i) Quantity of air

(ii) Heat supplied per kg of air circulated

(iii) Thermal efficiency of the cycle



T-s diagram

Solution:

For process 1-2 (Isentropic or reversible adiabatic compression)

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} = r_p^{\frac{\gamma-1}{\gamma}} \Rightarrow T_2 = T_1 r_p^{\frac{\gamma-1}{\gamma}}$$

$$\therefore T_2 = 291 \times (5)^{\frac{1.4-1}{1.4}} = 460.88K$$

Actual compressor outlet temperature, $T_{2'}$:

We know that Compressor Efficiency, η_c is given by

$$\eta_c = \frac{T_2 - T_1}{T_{2'} - T_1} \Rightarrow 0.85 = \frac{460.88 - 291}{T_{2'} - 291}$$

$$\therefore T_{2'} = 490.86K$$

Compressor work, w_c:

$$w_c = c_{p_{air}}\left(T_{2'} - T_1\right) = 1.005 \times 10^3 \times (490.86 - 291)$$

$$\therefore w_c = 200.86 \times 10^3 J/kg$$

For process 3'-4 (Isentropic or reversible adiabatic expansion)

$$\frac{T_{3'}}{T_4}=\left(\frac{p_{3'}}{p_4}\right)^{\left(\frac{\gamma-1}{\gamma}\right)_{gases}} \Rightarrow T_4=\frac{T_{3'}}{\left(\frac{p_{3'}}{p_4}\right)^{\left(\frac{\gamma-1}{\gamma}\right)_{gases}}}$$

$$\therefore T_4=\frac{951}{(4.9)^{\left(\frac{1.3-1}{1.3}\right)_{gases}}}=658.99K$$

Actual turbine outlet temperature, $T_{4'}$:

We know that turbine efficiency, η_t is given by

$$\eta_t=\frac{T_{3'}-T_{4'}}{T_{3'}-T_4}\Rightarrow 0.8=\frac{951-T_{4'}}{951-658.99}$$

$$\therefore T_{4'}=717.39K$$

Turbine work, w_t:

$$w_t=c_{p_{gases}}\left(T_{3'}-T_{4'}\right)=1.024\times10^3\times(951-717.39)$$

$$\therefore w_t=239.22\times10^3 J/kg$$

Net work done, w_{net}:

$$w_{net}=w_t-w_c=(239.22-200.86)\times10^3$$

$$\therefore w_{net}=38.36\times10^3 J/kg$$

Heat supplied, q_{sup}:

$$q_{sup}=\frac{c_{p_{gases}}\left(T_{3'}-T_{2'}\right)}{\eta_{comb}}=\frac{1.024\times10^3\times(951-490.86)}{0.85}$$

$$\therefore q_{sup}=554.33\times10^3 J/kg$$

Thermal efficiency of the cycle, η:

$$\eta=\frac{w_{net}}{q_{sup}}=\frac{38.36\times10^3}{554.33\times10^3}$$

$$\therefore \eta=0.1505(15.05\%)$$

Mass flow rate of air, m_a:

$$P = \dot{m}_a \times w_{net} \Rightarrow 1065 \times 10^3 = \dot{m}_a \times 83.45 \times 10^3$$

$$\therefore \dot{m}_a = 12.76 kg/s$$

1.19 Methods for improving work output (thermal efficiency) of Brayton cycle

The following are the three methods to improve the work output (or thermal efficiency) of a Brayton cycle:

(i) **Regeneration**

Utilisation of heat present in exhaust gases to pre-heat the air supplied to the combustion chamber.

(ii) **Intercooling**

Cooling the air compressed in first stage to the initial temperature before it is sent to the second stage compressor. This results in reducing the work input to the multi-stage compressor.

(iii) **Reheating**

Heating the air expanded in first stage to the maximum cycle temperature before it is sent to the second stage turbine. This results in increasing the work output of the multi-stage turbine.

1.20 Regeneration

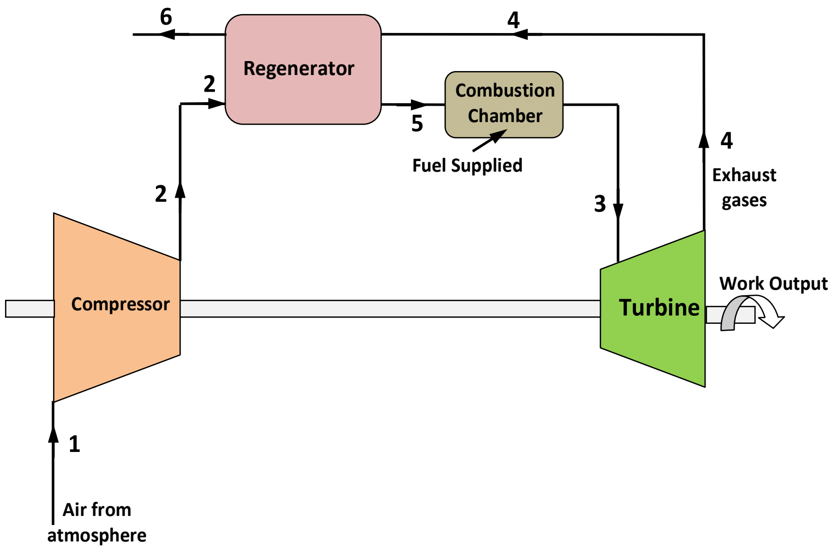
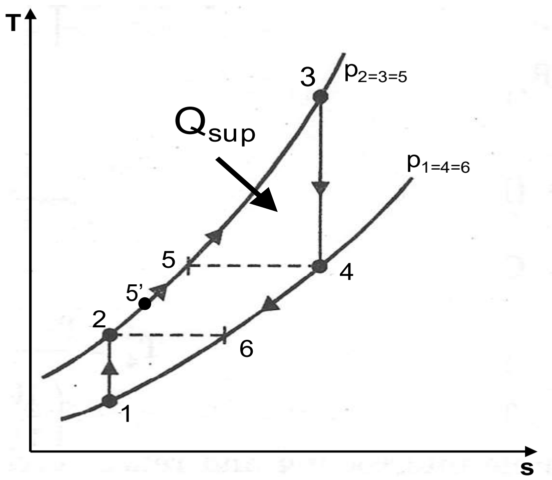


Fig. 1.18 Schematic diagram regeneration cycle

- A schematic diagram of regeneration cycle is shown in Fig. 1.18.
- Exhaust gases leaving the turbine have a higher temperature compared to that of the air leaving the compressor.
- The air leaving the compressor can be heated by hotter exhaust gases using a counter flow heat exchanger known as 'regenerator'.
- Thus, pre-heated air can be used in the combustion chamber, decreasing the heat input requirements (improves fuel economy) for the same net work output.



T-s diagram

- Ideally, the compressed air may be heated to the temperature (T_5) equal to that of the exhaust gases leaving the turbine (T_4). In other words, in a 100% efficient regenerator, the heat given by the exhaust gases equals the heat received by the compressed air.

- However, in practice, regenerator has less than 100% efficiency, resulting in the temperature of the compressed air increased to a value less than that of the exhaust gas temperature.
- Efficiency of regenerator is expressed in terms of effectiveness.
- Effectiveness is the ratio of actual heat transfer rate to the maximum possible heat transfer rate from the exhaust gases to the air.
- Effectiveness of regenerator:

$$\varepsilon = \frac{T_{5'} - T_2}{T_{5=4} - T_2} \quad where \;\; T_{5'} < T_{5=4}$$

- Lower pressure ratio and high cycle temperature are desirable for regeneration cycle as large heat recovery is possible. At higher pressure ratios, efficiency of cycle is higher without regeneration.
- Use of a regenerator improves the thermal efficiency as the heat addition in combustion chamber is reduced. However, the work output of the cycle remains unchanged.

1.21 Intercooling

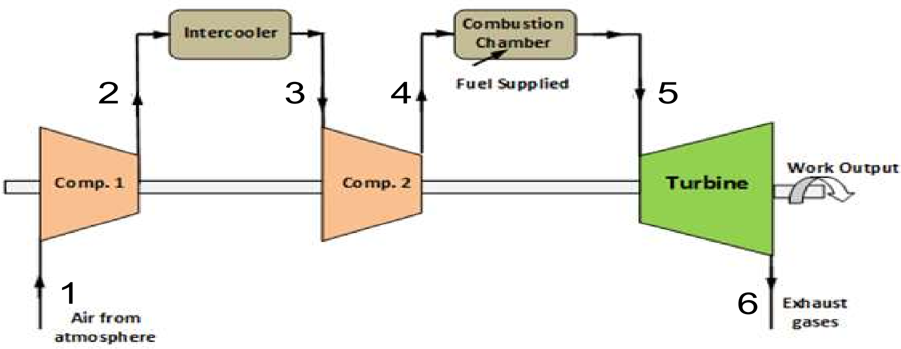


Fig. 1.19 Schematic diagram of intercooling cycle

- Fig. 1.19 shows a schematic diagram of intercooling cycle.
- Multi-stage compression of air to the required pressure ratio with intercooling reduces the work input to the compressor to a maximum extent.
- Thus, the net work output of the gas turbine plant is increased.

- The air, after first stage compression, is cooled in an ideal heat exchanger (intercooler) to the initial temperature of air ($T_3 = T_1$)
- For the same turbine work output, the work supplied to the compressor is reduced. Thus, the net work output of the cycle is increased.
- However, the heat supplied to the combustion chamber is increased. Hence, the thermal efficiency of the cycle is reduced.

1.21.1 Optimum Intercooling Pressure for Minimum Work Input

Referring to the T-s diagram, work input to two compressors with intercooling is given by

$$w_c = w_{c_1} + w_{c_2}$$

$$w_c = c_{p_a}\left(T_{2'} - T_1\right) + c_{p_a}\left(T_{4'} - T_3\right) \qquad \text{------ (12)}$$

Defining the isentropic efficiencies for the two compressors

$$\eta_{c_1} = \frac{T_2 - T_1}{T_{2'} - T_1} \; and \; \eta_{c_2} = \frac{T_4 - T_3}{T_{4'} - T_3} \qquad \text{------ (13)}$$

Substituting Eqn. (13) in (12), the work input to two compressors with intercooling is written as

$$w_c = c_{p_a}\frac{\left(T_2 - T_1\right)}{\eta_{c_1}} + c_{p_a}\frac{\left(T_4 - T_3\right)}{\eta_{c_2}} \qquad \text{------ (14)}$$

$$w_c = \frac{c_{p_a}T_1}{\eta_{c_1}}\left(\frac{T_2}{T_1} - 1\right) + \frac{c_{p_a}T_3}{\eta_{c_2}}\left(\frac{T_4}{T_3} - 1\right)$$

For the reversible adiabatic processes 1-2 and 3-4, we can write

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{p_i}{p_1}\right)^{\frac{\gamma-1}{\gamma}} \; and$$

$$\frac{T_4}{T_3} = \left(\frac{p_4}{p_3}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{p_2}{p_i}\right)^{\frac{\gamma-1}{\gamma}} \qquad \text{------ (15)}$$

where p_i is the intermediate pressure.

Substituting Eqn. (15) in (14),

$$w_c = \frac{c_{p_a} T_1}{\eta_{c_1}} \left[\left(\frac{p_i}{p_1} \right)^{\frac{\gamma-1}{\gamma}} - 1 \right] + \frac{c_{p_a} T_3}{\eta_{c_2}} \left[\left(\frac{p_2}{p_i} \right)^{\frac{\gamma-1}{\gamma}} - 1 \right] \qquad \text{------ (16)}$$

Let

$$x = \frac{\gamma - 1}{\gamma}$$

Then, Eqn. (16) becomes

$$w_c = \frac{c_{p_a} T_1}{\eta_{c_1}} \left[\left(\frac{p_i}{p_1} \right)^{x} - 1 \right] + \frac{c_{p_a} T_3}{\eta_{c_2}} \left[\left(\frac{p_2}{p_i} \right)^{x} - 1 \right] \qquad \text{------ (17)}$$

Differentiating the Eqn. (17) with respect to p_i and then equating it to zero, gives the optimum intercooler pressure for minimum work input.

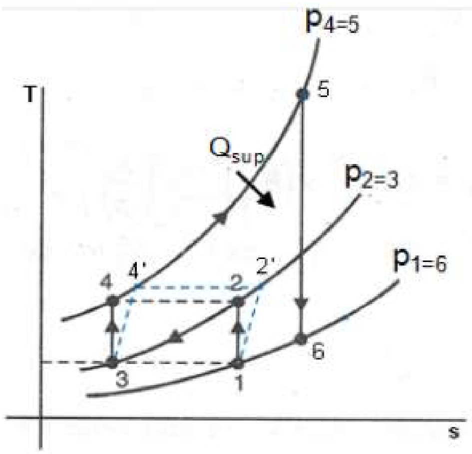
$$\frac{dw_c}{dp_i} = \frac{d}{dp_i} \left\{ \frac{c_{p_a} T_1}{\eta_{c_1}} \left[\left(\frac{p_i}{p_1} \right)^{x} - 1 \right] + \frac{c_{p_a} T_3}{\eta_{c_2}} \left[\left(\frac{p_2}{p_i} \right)^{x} - 1 \right] \right\} = 0$$

$$\Rightarrow \frac{dw_c}{dp_i} = \frac{T_1}{\eta_{c_1}} \left[\frac{x p_i^{x-1}}{p_1^x} - 0 \right] + \frac{T_3}{\eta_{c_2}} \left[p_2^x (-x) p_i^{-x-1} - 0 \right] = 0$$

$$\Rightarrow \frac{T_3}{T_1} = \frac{\eta_{c_2}}{\eta_{c_1}} \frac{p_i^{x-1}}{p_1^x} \frac{1}{p_2^x p_i^{-(x+1)}}$$

$$\Rightarrow \frac{T_3}{T_1} = \frac{\eta_{c_2}}{\eta_{c_1}} \frac{p_i^{2x}}{(p_1 p_2)^x}$$

$$\therefore p_i^{2x} = \frac{T_3}{T_1} \frac{\eta_{c_1}}{\eta_{c_2}} (p_1 p_2)^x \qquad \text{------ (18)}$$



T-s diagram

For $\eta_{c1} = \eta_{c2}$ and $T_3 = T_1$ (Perfect intercooling), Eqn. (18) becomes

$$p_i^{2x} = (p_1 p_2)^x$$

$$\therefore p_i = \sqrt{p_1 p_2} \Rightarrow \frac{p_i}{p_1} = \frac{p_2}{p_i}$$

That is, stage pressure ratios are equal for minimum work input.

1.22 Reheating

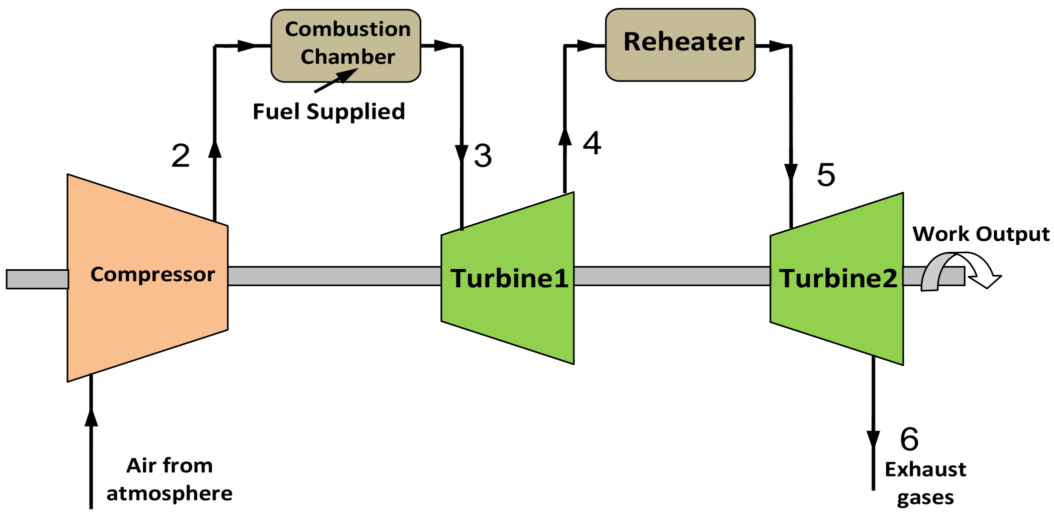


Fig. 1.20 Schematic diagram of reheating cycle

- The schematic diagram of a reheating cycle is shown in Fig. 1.20.
- The work output of a turbine operating between two pressure limits can be increased by expanding the gas in stages and reheating it in between.
- This is accomplished without raising the maximum temperature in the cycle.
- The reheating can be accomplished by spraying additional fuel into the oxygen-rich exhaust gases between two expansion states.
- Ideally, the reheating is done in such a way that the temperature of the gases after the first expansion is raised to that of the maximum cycle temperature.
- As the number of stages is increased, the expansion process becomes nearly isothermal.
- Although reheating results in increased specific work output, this gain is achieved with the expense of efficiency because of the additional heat supply.

1.22.1 Optimum reheat pressure for maximum work output

Referring to the T-s diagram, work done by two turbines with reheating is given by

$$w_t = w_{t_1} + w_{t_2} \qquad \text{------ (19)}$$

$$w_t = c_{p_g}\left(T_3 - T_{4'}\right) + c_{p_g}\left(T_5 - T_{6'}\right)$$

Defining the isentropic efficiencies for the two turbines

$$\eta_{t_1} = \frac{T_3 - T_{4'}}{T_3 - T_4}$$

and

$$\eta_{t_2} = \frac{T_5 - T_{6'}}{T_5 - T_6} \qquad \text{------ (20)}$$

Substituting Eqn. (20) in (19),

$$w_t = \eta_{t_1} c_{p_g}\left(T_3 - T_4\right) + \eta_{t_2} c_{p_g}\left(T_5 - T_6\right)$$

$$w_t = \eta_{t_1} c_{p_g} T_3\left(1 - \frac{T_4}{T_3}\right) + \eta_{t_2} c_{p_g} T_5\left(1 - \frac{T_6}{T_5}\right) \qquad \text{------ (21)}$$

For the reversible adiabatic processes 3-4 and 5-6, we can write

$$\frac{T_4}{T_3} = \left(\frac{p_4}{p_3}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{p_i}{p_2}\right)^{\frac{\gamma-1}{\gamma}} \; and$$

$$\frac{T_6}{T_5} = \left(\frac{p_6}{p_5}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{p_1}{p_i}\right)^{\frac{\gamma-1}{\gamma}} \qquad \text{------ (22)}$$

Let

$$x = \frac{\gamma - 1}{\gamma} \qquad \text{------ (23)}$$

Substituting Eqn. (22) and (23) in Eqn. (21)

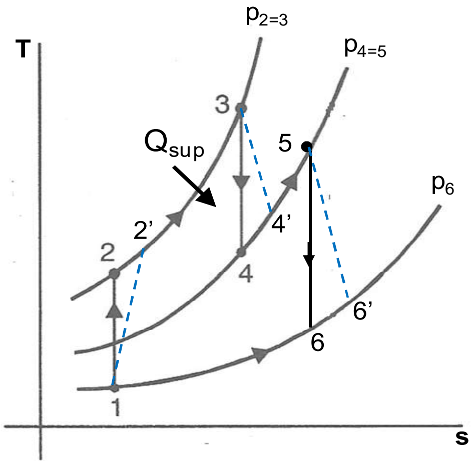
$$w_t = \eta_{t_1} c_{p_g} T_3\left[1 - \left(\frac{p_i}{p_2}\right)^x\right] + \eta_{t_2} c_{p_g} T_5\left[1 - \left(\frac{p_1}{p_i}\right)^x\right] \qquad \text{------ (24)}$$

Differentiating Eqn. (24) with respect to p_i and then equating it to zero, gives the optimum intermediate reheat pressure for maximum work output.

$$\frac{d}{dp_i}\left\{\eta_{t_1} c_{p_g} T_3\left[1 - \left(\frac{p_i}{p_2}\right)^x\right] + \eta_{t_2} c_{p_g} T_5\left[1 - \left(\frac{p_1}{p_i}\right)^x\right]\right\} = 0$$

$$\eta_{t_1} c_{p_g} T_3\left[0 - \frac{1}{p_2^x} x p_i^{x-1}\right] + \eta_{t_2} c_{p_g} T_5\left[0 - p_1^x\left(-x\right) p_i^{-x-1}\right] = 0$$

Simplifying,



T-s diagram

$$\eta_{t_1} T_3 \frac{1}{p_2^x} p_i^{x-1} = \eta_{t_2} T_5 p_1^x p_i^{-x-1}$$

$$\frac{p_i^{x-1}}{p_i^{-x-1}} = \frac{\eta_{t_2} T_5}{\eta_{t_1} T_3} p_1^x p_2^x$$

$$\Rightarrow \frac{p_i^{x-1}}{p_i^{-x-1}} = \frac{p_i^{x-1}}{p_i^{-(x+1)}} = \frac{\eta_{t_2} T_5}{\eta_{t_1} T_3} p_1^x p_2^x$$

$$\Rightarrow p_i^{x-1+x+1} = p_i^{2x} = \frac{\eta_{t_2} T_5}{\eta_{t_1} T_3} (p_1 p_2)^x$$

$$\therefore p_i^2 = \frac{\eta_{t_2} T_5}{\eta_{t_1} T_3} p_1 p_2 \qquad \text{------ (25)}$$

For $\eta_{t1} = \eta_{t2}$ and $T_3 = T_5$ (Perfect reheating), Eqn. (25) becomes

$$p_i = \sqrt{p_1 p_2}$$

That is,

$$\frac{p_i}{p_1} = \frac{p_2}{p_i};$$

Stage pressure ratios are equal.

1.23 Numerical examples on methods of improving Brayton cycle efficiency

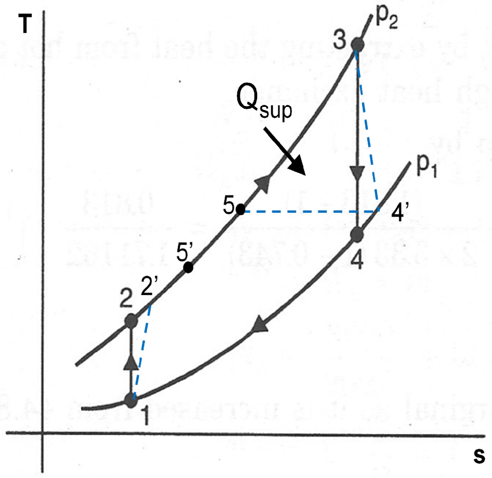
Numerical example 1.23.1

In a regenerative gas turbine cycle air enters the compressor at 1 bar 15^0C. Pressure ratio = 6. The isentropic efficiencies of compressor and turbine are respectively 0.8 and 0.85. The maximum temperature in the cycle is 800^0C. The regenerator efficiency = 0.78. Assume c = 1.1 kJ/kgK and γ = 1.32 for the combustion products. Find the efficiency of the cycle.

Data given:

- Regenerative Gas Turbine Cycle
- $p_1 = 1 \times 10^5$ N/m^2
- $T_1 = 15 + 273 = 288$ K

- $r_p = \frac{p_2}{p_1} = \frac{p_3}{p_4} = 6$
- $\eta_c = 0.8$
- $\eta_t = 0.85$
- $T_3 = 800 + 273 = 1073$ K
- $\varepsilon = 0.78$



T-s diagram

Given:

For gases:

- $c_p = 1.1$ kJ/kgK = 1.1×10^3 J/kgK
- $\gamma = 1.32$

Assume:

For air:

- $c_p = 1.005$ kJ/kgK = 1.005×10^3 J/kgK
- $\gamma = 1.4$

To determine:

Efficiency of the cycle

Solution:

Ideal (isentropic) temperature at compressor outlet, T_2:

We know that for process 1-2 (Isentropic or reversible adiabatic compression)

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} = r_p^{\frac{\gamma-1}{\gamma}} \Rightarrow T_2 = T_1 r_p^{\frac{\gamma-1}{\gamma}}$$

$$\therefore T_2 = 288 \times (6)^{\frac{1.4-1}{1.4}} = 480.52K$$

Actual temperature at compressor outlet, $T_{2'}$:

We know that compressor efficiency, η_c is given by

$$\eta_c = \frac{T_2 - T_1}{T_{2'} - T_1} \Rightarrow 0.8 = \frac{480.52 - 288}{T_{2'} - 288}$$
$$\therefore T_{2'} = 528.65K$$

Compressor work:

$$w_c = c_{p_{air}}\left(T_{2'} - T_1\right) = 1.005\times10^3 \times \left(528.65 - 288\right)$$
$$\therefore w_c = 241.85\times10^3 J/kg$$

Ideal (isentropic) temperature at outlet of turbine, T_4:

We know that for process 3-4 (Isentropic or reversible adiabatic expansion)

$$\frac{T_3}{T_4} = \left(\frac{p_3}{p_4}\right)^{\left(\frac{\gamma-1}{\gamma}\right)_{gases}} \Rightarrow T_4 = \frac{T_3}{\left(r_p\right)^{\left(\frac{\gamma-1}{\gamma}\right)_{gases}}}$$
$$\therefore T_4 = \frac{1073}{\left(6\right)^{\left(\frac{1.32-1}{1.32}\right)_{gases}}} = 694.99K$$

Actual temperature at turbine outlet, $T_{4'}$:

We know that turbine efficiency, η_t is given by

$$\eta_t = \frac{T_3 - T_{4'}}{T_3 - T_4} \Rightarrow 0.85 = \frac{1073 - T_{4'}}{1073 - 694.99}$$
$$\therefore T_{4'} = 751.69K$$

Turbine work, w_t:

$$w_t = c_{p_{gases}}\left(T_3 - T_{4'}\right) = 1.1\times10^3 \times \left(1073 - 751.69\right)$$
$$\therefore w_t = 353.44\times10^3 J/kg$$

Net work done, w_{net}:

$$w_{net} = w_t - w_c = \left(353.44 - 241.85\right)\times10^3$$
$$\therefore w_{net} = 111.59\times10^3 J/kg$$

Actual temperature at regenerator outlet, $T_{5'}$:

We know that regenerator effectiveness (efficiency), ε is given by

$$\varepsilon = \frac{T_{5'} - T_{2'}}{T_{5=4'} - T_{2'}} \Rightarrow 0.78 = \frac{T_{5'} - 528.65}{751.69 - 528.65}$$

$$\therefore T_{5'} = 702.62K$$

Heat supplied, q_{sup}:

$$q_{sup} = c_{p_{gases}}\left(T_3 - T_{5'}\right) = 1.1\times10^3 \times \left(1073 - 702.62\right)$$

$$\therefore q_{sup} = 407.42\times10^3 J/kg$$

Cycle efficiency, η:

$$\eta = \frac{w_{net}}{q_{sup}} = \frac{111.59}{407.42}$$

$$\therefore \eta = 0.2739(27.39\%)$$

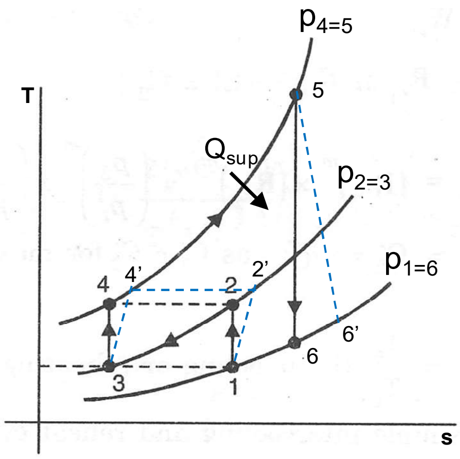
Numerical example 1.23.2

A gas turbine power plant consists of two compressors with perfect intercooling. The expansion occurs in a single turbine. The mass flow rate of air through the plant is 1 kg/min. Calorific value of the fuel used is 42000 kJ/kg, maximum and minimum temperatures in the cycle are 900^0C and 27^0C respectively. The working pressure limits are 1 bar and 6 bar respectively. The compressors have isentropic efficiencies of 80% and turbine 85%.The pressure ratios for both compressor stages are equal. Determine (i) overall efficiency of the plant (ii) output from the plant in kW (iii) air fuel ratio (iv) work ratio

Data given:

- Intercooling gas turbine cycle
- Perfect intercooling
- $\dot{m}_a$ = 1 kg/min
- CV = 42000 x 10^3 J/kg
- T_5 = 900 +273 = 1173 K
- T_1 = 27 +273 = 300 K

- $p_1 = 1 \times 10^5$ N/m^2
- $p_4 = 6 \times 10^5$ N/m^2
- $\eta_{c1} = \eta_{c2} = 0.8$
- $\eta_t = 0.85$
- $r_{pc1} = r_{pc2}$



T-s diagram

Assume:

For air:

- $c_p = 1.005$ kJ/kgK = 1.005×10^3 J/kgK
- $\gamma = 1.4$

To determine:

(i) Overall efficiency of the cycle

(ii) Output of plant in kW

(iii) Air-fuel ratio

(iv) Work ratio

Solution:

Intermediate pressure: $p_2 (= p_3)$

We know that for perfect intercooling:

$$p_2 = \sqrt{p_1 p_4} = \sqrt{1 \times 6}$$
$$\therefore p_2 = 2.45\ bar = 2.45 \times 10^5\ N/m^2$$

Ideal (isentropic) temperature at outlet of compressor-1, T_2:

We know that for process 1-2 (Isentropic or reversible adiabatic compression in comporessor-1)

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} = r_{p_{c1}}^{\frac{\gamma-1}{\gamma}} \Rightarrow T_2 = T_1 r_{p_{c1}}^{\frac{\gamma-1}{\gamma}}$$

$$\therefore T_2 = 300 \times (2.45)^{\frac{1.4-1}{1.4}} = 387.53K$$

Actual temperature at outlet of compressor-1, $T_{2'}$:

We know that efficiency of compressor-1, η_{c1} is given by

$$\eta_{c_1} = \frac{T_2 - T_1}{T_{2'} - T_1} \Rightarrow 0.8 = \frac{387.53 - 300}{T_{2'} - 300}$$

$$\therefore T_{2'} = 409.41K$$

Work of compressor-1, w_{c1}:

$$w_{c_1} = c_{p_{air}}\left(T_{2'} - T_1\right) = 1.005\times10^3 \times \left(409.41 - 300\right)$$

$$\therefore w_{c_1} = 109.96\times10^3 J/kg$$

Work of compressor-2, w_{c2}:

For minimum work input, perfect intercooling ($T_{2'} = T_{4'}$) and $\eta_{c2} = \eta_{c1}$, we have

$$T_3 = T_1 = 300K$$

$$T_4 = T_2 = 387.53K$$

$$T_{4'} = T_{2'} = 409.41K$$

$$\therefore w_{c_2} = w_{c_1} = 109.96\times10^3 J/kg$$

Total compressor work, w_c:

$$w_c = w_{c_1} + w_{c_2} = 2w_{c_1} \qquad \left\{\because w_{c_1} = w_{c2}\right\}$$

$$= 2\times109.96\times10^3 \Rightarrow \therefore w_c = 219.92\times10^3 J/kg$$

Ideal (isentropic) temperature at turbine outlet, T_6:

We know that for process 5-6 (Isentropic or reversible adiabatic expansion)

$$\frac{T_5}{T_6} = \left(r_{p_t}\right)^{\left(\frac{\gamma-1}{\gamma}\right)} \Rightarrow T_6 = \frac{T_5}{\left(r_{p_t}\right)^{\left(\frac{\gamma-1}{\gamma}\right)}} = \frac{1173}{\left(6\right)^{\frac{1.4-1}{1.4}}}$$

$$\therefore T_6 = 703.04K$$

Actual temperature at turbine outlet, $T_{6'}$:

We know that turbine efficiency, η_t is given by

$$\eta_t = \frac{T_5 - T_{6'}}{T_5 - T_6} \Rightarrow 0.85 = \frac{1173 - T_{6'}}{1173 - 703.04}$$

$$\therefore T_{6'} = 773.53K$$

Turbine work, w_t:

$$w_t = c_{p_{air}}(T_5 - T_{6'}) = 1.005\times10^3 \times (1173 - 773.53)$$
$$\therefore w_t = 401.47\times10^3 J/kg$$

Net work done, w_{net}:

$$w_{net} = w_t - w_c = (401.47 - 219.92)\times10^3$$
$$\therefore w_{net} = 181.55\times10^3 J/kg$$

Heat supplied, q_{sup}:

$$q_{sup} = q_{4'-5} = c_{p_{air}}(T_5 - T_{4'}) = 1.005\times10^3 \times (1173 - 409.41)$$
$$\therefore q_{4'-5} = 767.41\times10^3 J/kg$$

Overall efficiency of plant, η:

$$\eta = \frac{w_{net}}{q_{sup}} = \frac{181.55}{767.41} = 0.2366(23.66\%)$$

Output of plant in kW, P:

$$P = \dot{m}_a\, w_{net} = \frac{1}{60}\times181.55 = 3.03kW$$

Air-fuel Ratio, A/F:

We know that heat supplied by the fuel, q_{sup}, is given by

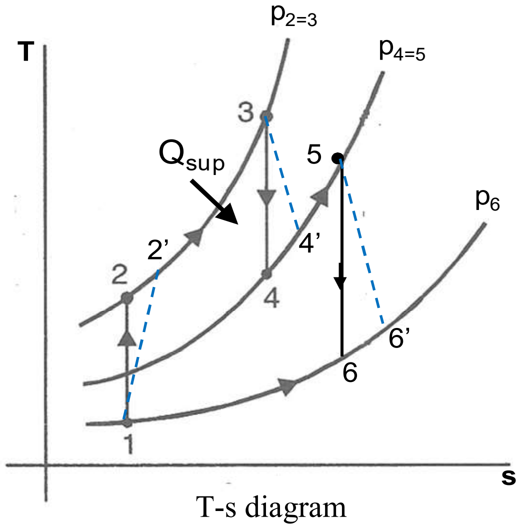
$$q_{sup} = m_f \times CV \Rightarrow 767.41\times10^3 = m_f \times 42000\times10^3$$
$$\therefore m_f = 0.0183$$
$$\therefore A/F = \frac{1}{0.0183} = 54.73:1$$

Work ratio, WR:

$$WR = \frac{w_{net}}{w_t} = \frac{181.55}{401.47} = 0.4522$$

Numerical example 1.23.3

In a reheat gas turbine cycle, comprising one compressor and two turbines, air is compressed from 1 bar at 27^{0}C to 6 bar. The highest temperature in the cycle is 900^{0}C. The expansion in the first stage turbine is such that the work from it just equals the work required by the compressor. Air is reheated between the two stages of expansion to 850^{0}C. Assume that the isentropic efficiencies of the compressor, the first stage and the second stage turbines are 85% each and that the working substance is air. Calculate the cycle efficiency.



T-s diagram

Data given:

- Reheat Gas Turbine Cycle
- $p_1 = 1 \times 10^5$ N/m^2
- $T_1 = 27 + 273 = 300$ K
- $r_p = \frac{p_2}{p_1} = \frac{p_3}{p_6} = 6$
- $T_3 = 900+273 = 1173$ K
- $w_{t1} = w_c$
- $T_5 = 850+273 = 1123$K
- $\eta_c = \eta_{t1} = \eta_{t2} = 0.85$

To determine:

- Cycle efficiency

Assume for air:

- $c_p = 1.005$ kJ/kgK $= 1.005 \times 10^3$ J/kgK
- $\gamma = 1.4$

Solution:

Ideal (isentropic) temperature at compressor outlet, T_2:

We know that for process 1-2 (Isentropic or reversible adiabatic compression)

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} = r_p^{\frac{\gamma-1}{\gamma}} \Rightarrow T_2 = T_1 r_p^{\frac{\gamma-1}{\gamma}}$$

$$\therefore T_2 = 300 \times (6)^{\frac{1.4-1}{1.4}} = 500.54K$$

Actual temperature at compressor outlet, $T_{2'}$:

We know that compressor efficiency, η_c is given by

$$\eta_c = \frac{T_2 - T_1}{T_{2'} - T_1} \Rightarrow 0.85 = \frac{500.54 - 300}{T_{2'} - 300}$$

$$\therefore T_{2'} = 535.93K$$

Compressor work, w_c:

$$w_c = c_{p_{air}}\left(T_{2'} - T_1\right) = 1.005 \times 10^3 \times (535.93 - 300)$$

$$\therefore w_c = 237.11 \times 10^3 J/kg$$

Actual temperature at turbine outlet, $T_{4'}$:

We know that the work of turbine-1, w_{t1}, is given by

$$w_{t_1} = c_{p_{air}}\left(T_3 - T_{4'}\right) \quad where \; w_{t_1} = w_c = 237.11 \times 10^3 J/kg$$

$$\Rightarrow 237.11 \times 10^3 = 1.005 \times 10^3 \times \left(1173 - T_{4'}\right)$$

$$\therefore T_{4'} = 937.07K$$

Ideal (isentropic) temperature at outlet of turbine-1, T_4:

We know that efficiency of turbine-1, η_t, is given by

$$\eta_{t_1} = \frac{T_3 - T_{4'}}{T_3 - T_4} \Rightarrow 0.85 = \frac{1173 - 937.07}{1173 - T_4}$$

$$\therefore T_4 = 895.44K$$

Pressure ratio of turbine-2: r_{pt2}

$$r_{p_{t2}} = \frac{p_5}{p_6} = \frac{r_p}{r_{p_{t1}}} = \frac{6}{2.57} = 2.33$$

Ideal (isentropic) temperature at outlet of turbine-2, T_6:

We know that for process 5-6 (Isentropic or reversible adiabatic expansion in turbine-2)

$$\frac{T_5}{T_6} = \left(r_{p_{t2}}\right)^{\left(\frac{\gamma-1}{\gamma}\right)} \Rightarrow T_6 = \frac{T_5}{\left(r_{p_{t2}}\right)^{\left(\frac{\gamma-1}{\gamma}\right)}} = \frac{1123}{\left(2.33\right)^{\frac{1.4-1}{1.4}}}$$

$$\therefore T_6 = 881.91K$$

Actual temperature at outlet of turbine-2, $T_{6'}$:

We know that efficiency of turbine-2, η_{t2}, is given by

$$\eta_{t_2} = \frac{T_5 - T_{6'}}{T_5 - T_6} \Rightarrow 0.85 = \frac{1123 - T_{6'}}{1123 - 881.91}$$

$$\therefore T_{6'} = 918.07K$$

Work of turbine-2, w_{t2}:

$$w_{t_2} = c_{p_{air}}\left(T_5 - T_{6'}\right) = 1.005\times10^3\times\left(1123 - 918.07\right)$$

$$\therefore w_{t_2} = 205.95\times10^3 J/kg$$

Total turbine work, w_t:

$$w_t = w_{t_1} + w_{t_2} = \left(237.11 + 205.95\right)\times10^3$$

$$\therefore w_t = 443.06\times10^3 J/kg$$

Net work done, w_{net}:

$$w_{net} = w_t - w_c = \left(443.06 - 237.11\right)\times10^3$$

$$\therefore w_{net} = 205.95\times10^3 J/kg$$

Heat supplied: $q_{sup} = q_{2'-3} + q_{4'-5}$:

$$q_{2'-3} = c_{p_{air}}\left(T_3 - T_{2'}\right) = 1.005\times10^3\times\left(1173 - 535.93\right)$$

$$\therefore q_{2'-3} = 640.26K$$

$$q_{4'-5} = c_{p_{air}}\left(T_5 - T_{4'}\right) = 1.005\times10^3\times\left(1123 - 937.07\right)$$

$$\therefore q_{4'-5} = 186.86\times10^3 J/kg$$

$$q_{sup} = q_{2'-3} + q_{4'-5} = \left(640.26 + 186.86\right)\times10^3$$

$$\therefore q_{sup} = 827.12\times10^3 J/kg$$

Cycle efficiency, η:

$$\eta = \frac{w_{net}}{q_{\sup}} = \frac{205.95}{827.12}$$

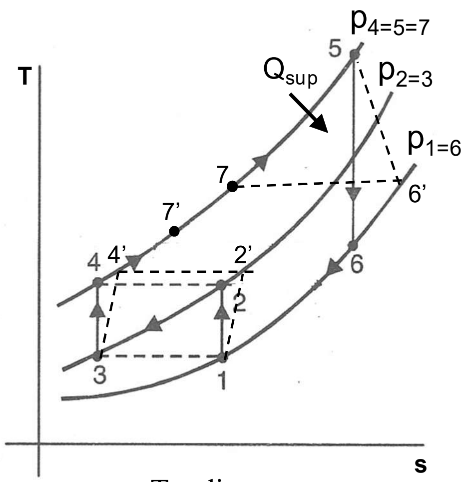
$$\therefore \eta = 0.249(24.9\%)$$

Numerical example 1.23.4

The total pressure ratio of 2-stage compressor with an intercooler is 8:1. Air enters at 1 bar and 20^{0}C. The maximum cycle temperature is limited to 600^{0}C. A regenerator is used to recover 85% of the available heat. The plant uses only one turbine for expansion. Assuming perfect intercooling and isentropic efficiencies of compressor and turbine as 0.85 and 0.8 respectively determine the plant efficiency and work ratio.

Data given:

- Intercooler and regenerator gas turbine cycle
- $p_1 = 1 \times 10^5$ N/m^2
- $T_1 = 20 + 273 = 293$ K
- $r_p =$ = 8
- $T_5 = 600 + 273 = 873$ K
- ε = 0.85
- Perfect intercooling
- $\eta_{c1} = \eta_{c2} = 0.85$
- $\eta_t = 0.8$



T-s diagram

Assume for air:

- $c_p = 1.005$ kJ/kgK = 1.005×10^3 J/kgK
- $\gamma = 1.4$

To determine:

(i) Plant efficiency

(ii) Work ratio

Solution:

Intermediate pressure: p_2

We know that for perfect intercooling:

$$p_2 = \sqrt{p_1 p_4} = \sqrt{1\times 8}$$

$$\therefore p_2 = 2.83\ bar = 2.83\times 10^5\ N/m^2$$

Ideal (isentropic) temperature at outlet of compressor-1, T_2:

We know that for process 1-2 (Isentropic or reversible adiabatic compression in compressor-1)

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} = r_{p_1}^{\frac{\gamma-1}{\gamma}} \Rightarrow T_2 = T_1 r_{p_1}^{\frac{\gamma-1}{\gamma}}$$

$$\therefore T_2 = 293\times(2.83)^{\frac{1.4-1}{1.4}} = 394.41K$$

Actual temperature at outlet of compressor-1, $T_{2'}$:

We know that efficiency of compressor-1, η_{c1}, is given by

$$\eta_{c_1} = \frac{T_2 - T_1}{T_{2'} - T_1} \Rightarrow 0.85 = \frac{394.41-293}{T_{2'}-293}$$

$$\therefore T_{2'} = 412.31K$$

Work of compressor-1, w_{c1}:

$$w_{c_1} = c_{p_{air}}\left(T_{2'} - T_1\right) = 1.005\times 10^3 \times (412.31-293)$$

$$\therefore w_{c_1} = 119.91\times 10^3\ J/kg$$

Work of compressor-2, w_{c2}:

For perfect intercooling ($T_{4'} = T_{2'}$) and $\eta_{c2} = \eta_{c1}$:

$$T_3 = T_1 = 293K$$

$$T_4 = T_2 = 394.01K$$

$$T_{4'} = T_{2'} = 412.31K$$

$$w_{c_1} = w_{c_2} = 119.91\times 10^3\ J/kg$$

Total compressor work, w_c:

$$w_c = w_{c_1} + w_{c_2} = 2 \times w_{c_1} \qquad \{\because w_{c_1} = w_{c_2}\}$$
$$= 2 \times 119.91 \times 10^3$$
$$\therefore w_c = 239.82 \times 10^3 \, J/kg$$

Ideal (isentropic) temperature at turbine outlet, T_6:

We know that for process 5-6 (Isentropic or reversible adiabatic expansion)

$$\frac{T_5}{T_6} = \left(r_p\right)^{\left(\frac{\gamma-1}{\gamma}\right)} \Rightarrow T_6 = \frac{T_5}{\left(r_p\right)^{\left(\frac{\gamma-1}{\gamma}\right)}} = \frac{873}{(8)^{\frac{1.4-1}{1.4}}}$$
$$\therefore T_6 = 481.95K$$

Actual temperature at turbine outlet, $T_{6'}$:

We know that turbine efficiency, η_t, is given by

$$\eta_t = \frac{T_5 - T_{6'}}{T_5 - T_6} \Rightarrow 0.80 = \frac{873 - T_{6'}}{873 - 481.95}$$
$$\therefore T_{6'} = 560.16K$$

Turbine work, w_t :

$$w_t = c_{p_{air}}\left(T_5 - T_{6'}\right) = 1.005 \times 10^3 \times \left(873 - 560.16\right)$$
$$\therefore w_{t_2} = 314.40 \times 10^3 \, J/kg$$

Net work done, w_{net}:

$$w_{net} = w_t - w_c = \left(314.40 - 239.82\right) \times 10^3$$
$$\therefore w_{net} = 74.58 \times 10^3 \, J/kg$$

Actual temperature at regenerator outlet, $T_{7'}$:

We know that regenerator effectiveness, ε, is given by

$$\varepsilon = \frac{T_{7'} - T_{4'}}{T_{7=6'} - T_{4'}} \Rightarrow 0.85 = \frac{T_{7'} - 411.84}{560.16 - 411.84}$$
$$\therefore T_{7'} = 537.91K$$

Heat supplied, q_{sup}:

$$q_{\sup} = c_{p_{air}}\left(T_5 - T_{7'}\right) = 1.005\times10^3\times\left(873-537.91\right)$$

$$\therefore q_{\sup} = 336.77\times10^3\, J/kg$$

Cycle efficiency, η:

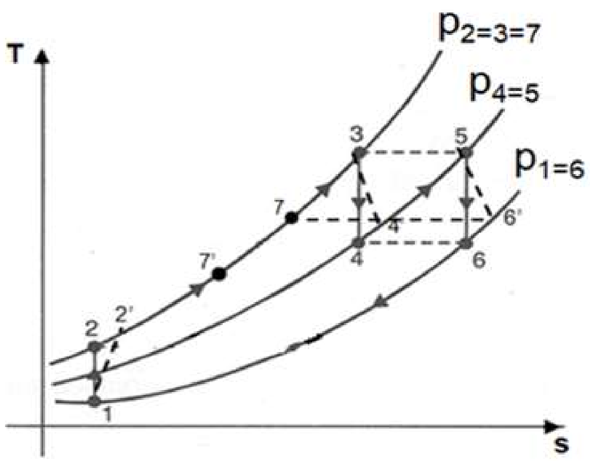
$$\eta = \frac{w_{net}}{q_{\sup}} = \frac{74.58}{336.77} \Rightarrow \therefore \eta = 0.2215(22.15\%)$$

Numerical example 1.23.5

In a gas turbine plant, air enters the compressor at 20^0C with a pressure ratio of 6.5. Then the air is heated to a maximum permissible temperature of 800^0C, first in a heat exchanger which is 70% efficient and then in the combustion chamber. The air is expanded in 2 stages such that the expansion work is maximum. The air is reheated to 800^0C after first stage expansion. Determine the thermal efficiency of the cycle, work ratio and net shaft work per kg of air. The isentropic efficiencies of compressor and turbine are 82% and 85% respectively.

Data given:

- Regenerator and reheater gas turbine cycle
- T_1 = 20 +273 = 293 K
- r_p = = 6.5
- T_3 = 800 +273 = 1073 K
- ε = 0.7
- T_5 = T_3 =800 +273 = 1073 K
- η_c = 0.82
- η_{t1} = η_{t2} = 0.85



T-s diagram

Assume for air:

- c_p = 1.005 kJ/kgK = 1.005 x 10^3 J/kgK
- γ = 1.4

To determine:

(i) Cycle efficiency

(ii) Work ratio

(iii) Net shaft work per kg

Solution:

Ideal (isentropic) temperature at compressor outlet, T_2:

We know that for process 1-2 (Isentropic or reversible adiabatic compression)

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} = r_p^{\frac{\gamma-1}{\gamma}} \Rightarrow T_2 = T_1 r_p^{\frac{\gamma-1}{\gamma}}$$

$$\therefore T_2 = 293 \times (6.5)^{\frac{1.4-1}{1.4}} = 500.17K$$

Actual temperature at compressor outlet, $T_{2'}$:

We know that compressor efficiency, η_c. is given by:

$$\eta_c = \frac{T_2 - T_1}{T_{2'} - T_1} \Rightarrow 0.82 = \frac{500.17 - 293}{T_{2'} - 293}$$

$$\therefore T_{2'} = 545.65K$$

Compressor work. w_c:

$$w_c = c_{p_{air}}\left(T_{2'} - T_1\right) = 1.005 \times 10^3 \times (545.65 - 293)$$

$$\therefore w_c = 253.91 \times 10^3 J/kg$$

Intermediate pressure: p_5 (= p_4)

We know that for maximum work output and perfect reheating

$$p_{5=4} = \sqrt{p_1 p_2} = \sqrt{1 \times 6.5}$$

$$\therefore p_{5=4} = 2.55\ bar = 2.55 \times 10^5 N/m^2$$

Ideal (isentropic) temperature at outlet of turbine-1, T_4:

We know that for process 3-4 (Isentropic or reversible adiabatic expansion in turbine-1)

$$\frac{T_3}{T_4} = \left(\frac{p_{3=2}}{p_4}\right)^{\left(\frac{\gamma-1}{\gamma}\right)} \Rightarrow T_4 = \frac{T_3}{\left(\frac{p_{3=2}}{p_4}\right)^{\left(\frac{\gamma-1}{\gamma}\right)}} = \frac{1073}{(2.55)^{\frac{1.4-1}{1.4}}}$$

$$\therefore T_4 = 821.20K$$

Actual temperature at outlet of turbine-1, $T_{4'}$:

We know that efficiency of turbine-1, η_{t1}, is given by:

$$\eta_{t_1} = \frac{T_3 - T_{4'}}{T_3 - T_4} \Rightarrow 0.85 = \frac{1073 - T_{4'}}{1073 - 821.20}$$

$$\therefore T_{4'} = 858.97K$$

Work of turbine-1, w_{t1}:

$$w_{t_1} = c_{p_{air}}\left(T_3 - T_{4'}\right) = 1.005 \times 10^3 \times (1073 - 858.97)$$

$$\therefore w_{t_1} = 215.10 \times 10^3 J/kg$$

Work of turbine-2, w_{t2}:

We know that for maximum work output, perfect reheating and $\eta_{t2} = \eta_{t1}$:

$$T_3 = T_5 = 1073K$$

$$T_6 = T_4 = 821.20K$$

$$T_{6'} = T_{4'} = 858.97K$$

$$\therefore w_{t_2} = w_{t_1} = 215.10 \times 10^3 J/kg$$

Total work of turbine, w_t:

$$w_t = w_{t_1} + w_{t_2} = 2 \times w_{t_1} \qquad \left\{\because w_{t_1} = w_{t_2}\right\}$$

$$= 2 \times 215.10 \times 10^3$$

$$\therefore w_t = 430.20 \times 10^3 J/kg$$

Net work done (net shaft work per kg), w_{net}:

$$w_{net} = w_t - w_c = (430.20 - 253.91) \times 10^3$$

$$\therefore w_{net} = 176.29 \times 10^3 J/kg$$

Actual temperature at regenerator outlet, T7':

We know that regenerator effectiveness, ε, is given by

$$\varepsilon = \frac{T_{7'} - T_{2'}}{T_{7=4'=6'} - T_{2'}} \Rightarrow 0.70 = \frac{T_{7'} - 545.65}{858.97 - 545.65}$$

$$\therefore T_{7'} = 764.97K$$

Heat supplied, q_{sup}:

$$q_{sup} = c_{p_{air}}\left(T_3 - T_{7'}\right) + c_{p_{air}}\left(T_5 - T_{4'}\right)$$

$$= 1.005\times10^3 \times\left(1073 - 764.97\right) + 1.005\times10^3 \times\left(1073 - 858.97\right)$$

$$\therefore q_{sup} = 524.67\times10^3 J/kg$$

Cycle efficiency, η:

$$\eta = \frac{w_{net}}{q_{sup}} = \frac{176.29}{524.67} \Rightarrow \therefore \eta = 0.3360(33.60\%)$$

Work ratio, WR:

$$WR = \frac{w_{net}}{w_t} = \frac{176.29}{430.20}$$

$$\therefore WR = 0.4098$$

Numerical example 1.23.6

Consider an ideal gas-turbine cycle with 2 stages of compression and 2 stages of expansion. The pressure ratio across each stage of compressor and turbine is 3. The initial temperature of air entering the compressor is 27^0C. The air enters each stage of the turbine at 1200K. Determine the work ratio and the thermal efficiency of the cycle, when no regenerator is used. Assume an efficiency of 80% for each compressor stage and an efficiency of 85% for each turbine stage.

Data given:

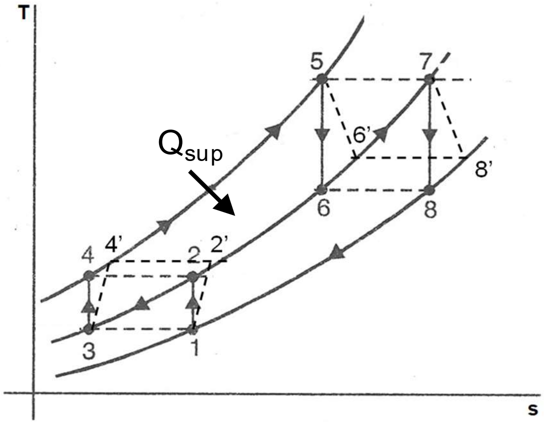
- Intercooler and reheater gas turbine cycle
- $r_p = 3$

- $T_1 = 27 + 273 = 300$ K
- $T_5 = T_7 = 1200$ K
- $\eta_{c1} = \eta_{c2} = 0.80$
- $\eta_{t1} = \eta_{t2} = 0.85$

To determine:

(i) Work ratio

(ii) Thermal efficiency



T-s diagram

Assume for air:

- $c_p = 1.005$ kJ/kgK $= 1.005 \times 10^3$ J/kgK
- $\gamma = 1.4$
- Perfect Intercooling, Minimum Work Input
- Perfect Reheating, Maximum Work Output

Solution:

Ideal (isentropic) temperature at outlet of compressor-1, T_2:

We know that for process 1-2 (Isentropic or reversible adiabatic compression in compressor-1)

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} = r_{p_{c_1}}^{\frac{\gamma-1}{\gamma}} \Rightarrow T_2 = T_1 r_{p_{c_1}}^{\frac{\gamma-1}{\gamma}}$$

$$\therefore T_2 = 300 \times (3)^{\frac{1.4-1}{1.4}} = 410.62K$$

Actual temperature at outlet of compressor-1, $T_{2'}$:

We know that efficiency of compressor-1, η_{c1}, is given by

$$\eta_{c_1} = \frac{T_2 - T_1}{T_{2'} - T_1} \Rightarrow 0.80 = \frac{410.62 - 300}{T_{2'} - 300}$$

$$\therefore T_{2'} = 438.28K$$

Work of compressor-1, w_{c1} :

$$w_{c_1} = c_{p_{air}}\left(T_{2'} - T_1\right) = 1.005 \times 10^3 \times (438.28 - 300)$$

$$\therefore w_{c_1} = 138.97 \times 10^3 J/kg$$

Work of compressor-2, w_{c2}:

We know that for minimum work input, perfect intercooling and $\eta_{c2} = \eta_{c1}$:

$$T_3 = T_1 = 300K$$
$$T_2 = T_4 = 410.62K$$
$$T_{2'} = T_{4'} = 438.28K$$
$$\therefore w_{c_2} = w_{c_1} = 138.97 \times 10^3 J/kg$$

Total compressor work, w_c:

$$w_c = 2w_{c_1} = 2 \times 138.97 \times 10^3 = 277.94 \times 10^3 J/kg$$

Ideal (isentropic) temperature at outlet of turbine-1, T_6:

We know that for process 5-6 (Isentropic or reversible adiabatic expansion in turbine-1)

$$\frac{T_5}{T_6} = \left(r_{p_{t_1}}\right)^{\left(\frac{\gamma-1}{\gamma}\right)} \Rightarrow T_6 = \frac{T_5}{\left(r_{p_{t_1}}\right)^{\left(\frac{\gamma-1}{\gamma}\right)}} = \frac{1200}{(3)^{\frac{1.4-1}{1.4}}}$$
$$\therefore T_6 = 876.73K$$

Actual temperature at outlet of turbine-1, $T_{6'}$:

We know that efficiency of turbine-1, η_{t1}, is given by

$$\eta_{t_1} = \frac{T_5 - T_{6'}}{T_5 - T_6} \Rightarrow 0.85 = \frac{1200 - T_{6'}}{1200 - 876.73}$$
$$\therefore T_{6'} = 925.22K$$

Work of turbine-1, w_{t1} :

$$w_{t_1} = c_{p_{air}}\left(T_5 - T_{6'}\right) = 1.005 \times 10^3 \times \left(1200 - 925.22\right)$$
$$\therefore w_{t_1} = 276.15 \times 10^3 J/kg$$

Work of turbine-2, w_{t2}:

We know that for maximum work output, perfect reheating and $\eta_{t2} = \eta_{t1}$:

$$T_5 = T_7 = 1200K$$
$$T_6 = T_8 = 876.73K$$
$$T_{6'} = T_{8'} = 925.22K$$
$$\therefore w_{t_2} = w_{t_1} = 276.15 \times 10^3 J/kg$$

Total work of turbine, w_t:

$$w_t = 2w_{t_1} = 2\times 276.15 = 552.3\times 10^3\, J/kg$$

Net work done, w_{net} :

$$w_{net} = w_t - w_c = (552.3 - 277.94)\times 10^3$$
$$\therefore w_{net} = 274.36\times 10^3\, J/kg$$

Heat supplied, q_{sup} :

$$q_{sup} = c_{p_{air}}(T_5 - T_{4'}) = 1.005\times 10^3 \times (1200 - 438.28)$$
$$\therefore q_{sup} = 765.53\times 10^3\, J/kg$$

Thermal efficiency, η:

$$\eta = \frac{w_{net}}{q_{sup}} = \frac{274.36}{765.53} = 0.3584(35.84\%)$$

Work ratio, WR:

$$WR = \frac{w_{net}}{w_t} = \frac{w_{net}}{2w_{t_1}} = \frac{274.36}{2\times 276.15} = 0.4968$$

Numerical example 1.23.7

(Reconsider the Numerical Example (1.23.6) with a regenerator with an effectiveness of 0.75. Compare the result)

Data given:

- Intercooler and reheater gas turbine cycle with a regenerator
- $r_p = 3$
- $T_1 = 27 + 273 = 300$ K
- $T_5 = T_7 = 1200$ K
- $\eta_{c1} = \eta_{c2} = 0.80$
- $\eta_{t1} = \eta_{t2} = 0.85$
- $\varepsilon = 0.75$

To determine:

(i) Work Ratio

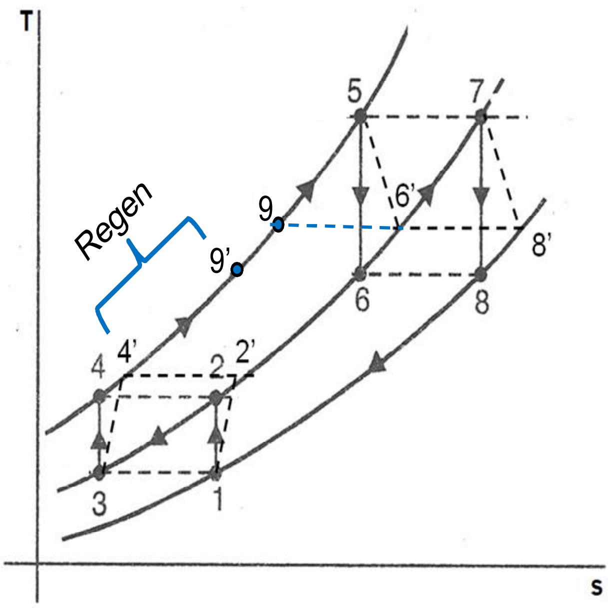
(ii) Thermal Efficiency

Solution:

From example (1.23.6):

We have the following results:

$$T_{2=4} = 410.62K, T_{2'=4'} = 438.28K$$
$$T_{6=8} = 876.73K, T_{6'=8'} = 925.22K$$
$$w_{net} = 274.36 \times 10^3 J/kg$$



T-s diagram

Actual temperature at regenerator outlet, $T_{9'}$:

We know that regenerator effectiveness, ε, is given by

$$\varepsilon = \frac{T_{9'} - T_{4'}}{T_{9=6'=8'} - T_{4'}}$$
$$0.75 = \frac{T_{9'} - 438.28}{925.22 - 438.28}$$
$$\therefore T_{9'} = 803.49K$$

Heat supplied: q_{sup}

$$q_{sup} = c_{p_{air}}\left(T_5 - T_{9'}\right) = 1.005 \times 10^3 \times \left(1200 - 803.49\right)$$
$$\therefore q_{sup} = 396.51 \times 10^3 J/kg$$

Work ratio, WR:

No change in work ratio, because net work done and turbine work remains the same.

Thermal efficiency, η:

$$\eta = \frac{w_{net}}{q_{sup}} = \frac{274.36}{396.51} = 0.6919\left(69.19\%\right)$$

Percentage increase in thermal efficiency:

$\%\ increase\ in\ thermal\ efficiency$

$$= \frac{\eta_{with\ regen} - \eta}{\eta} \times 100$$

$$= \frac{69.19 - 35.84}{35.84} \times 100 = 93.05\%$$

1.24 Introduction to the principles of jet propulsion

- Gas turbine cycles used to power aircraft engines are known as "jet-propulsion" cycles.
- In an ideal jet-propulsion cycle, the gases are not expanded to the ambient pressure in the turbine, as in the case of an ideal Brayton cycle.
- Instead, the gases are expanded to a pressure such that the power produced by the turbine is just sufficient to drive the compressor and the auxiliary equipment (such as small generator, pump, etc.,)
- Thus, the net work output of a jet-propulsion cycle is zero.
- The gases coming out of the turbine at high pressure are subsequently accelerated in a nozzle to provide the thrust to propel the aircraft.

1.25 Classification of gas turbine engines for jet propulsion

The thrust force required for propulsion can be obtained by either propelling a large mass of air or by increasing the velocity of a small mass of air. Based on this principle, gas turbine engines for jet propulsion are classified as:

- Turbo-jet Engine
    - Turbo-prop Engine
    - Turbo-fan Engine
- Ram-jet engine
    - Pulse-jet engine

1.26 Ideal turbojet cycle

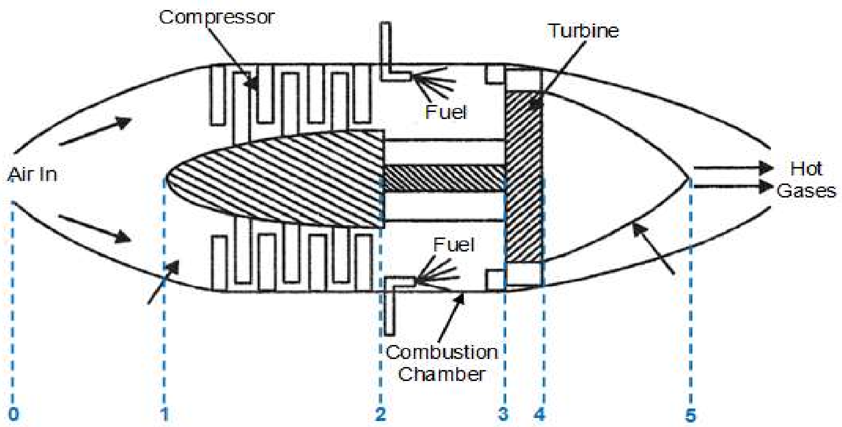
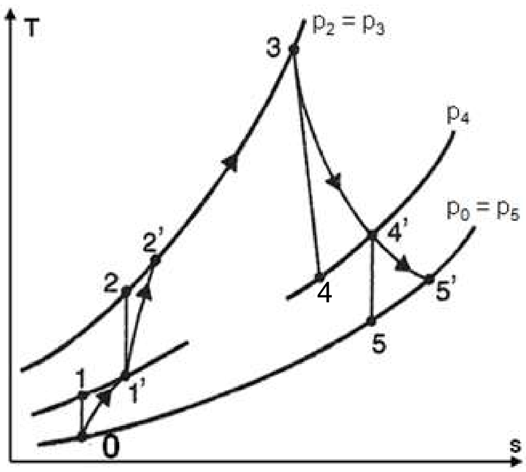


Fig. 1.21 Schematic diagram of an ideal turbojet engine

- The schematic diagram of an ideal turbojet engine is shown in Fig. 1.21.
- In this engine, a small mass of air is taken in whose velocity is increased to a larger value.
- The pressure of incoming air rises slightly as it is decelerated in diffuser. This reduces the work required to run the compressor.
- Air is compressed further by the compressor to the required pressure.
- It is then mixed with fuel in the combustion chamber, where the mixture is burned at constant pressure.



T-s diagram

- The high-pressure and high-temperature combustion gases partially expand in the turbine, producing enough power to drive the compressor and other equipment.
- Finally, the gases expand in a nozzle to the ambient pressure and leave the engine at a high velocity.

- The high velocity jet of hot gases provides thrust to the aircraft to give the forward motion by jet reaction.

1.26.1 Advantages and Disadvantages of turbo-jet engine

Advantages:

- Simple in construction and easy to maintain.
- Occupies less space.
- Attain higher flight speeds (800 to 1000 kmph)
- Rate of climb is higher.
- A wide variety of fuels can be used.
- Performance is not affected very much by A/F ratio used.

Disadvantages:

- Materials used are costlier and have shorter life.
- Produces more noise.
- Requires a long run-way, if the take-off period is higher.
- Specific fuel consumption is higher at low altitudes.

1.27 Turboprop (Propjet) engine

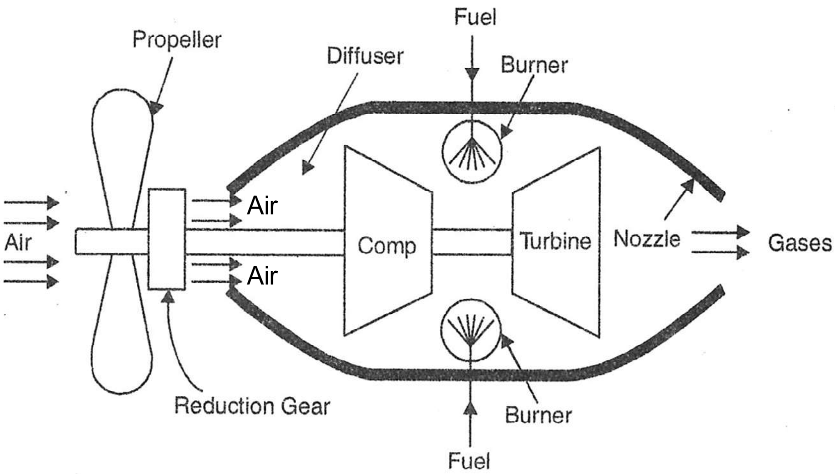


Fig. 1.22 Schematic diagram of a turboprop engine

- A turboprop engine is shown schematically in Fig. 1.22.
- A turbo-prop engine differs from turbo-jet engine in that it uses a propeller to increase the mass flow of air entering the diffuser. This increases the higher thrust per mass flow of fuel and also results in better fuel economy.
- The working of the engine is similar to a turbo-jet engine.
- Nearly 80% the expansion takes place in the turbine in order to run the compressor and propeller.
- The gas turbine speed being very high, a reduction gear is used to run the propeller.

1.27.1 Advantages and disadvantages of turbo-prop engine

Advantages:

- Provides higher thrust at take-off and better fuel economy.
- Operates most economically over a wide range of speed (400 to 800 kmph).
- Maintenance is easy and it has lower vibration and noise.
- The drag is less because of less frontal area.

Disadvantages:

- Lower propeller efficiency at higher speeds due to shock and flow separation.
- Reduction gear is essential which increases maintenance cost and loss of energy due to gear friction.

1.28 Turbofan (Fanjet or Bypass Turbojet) engine

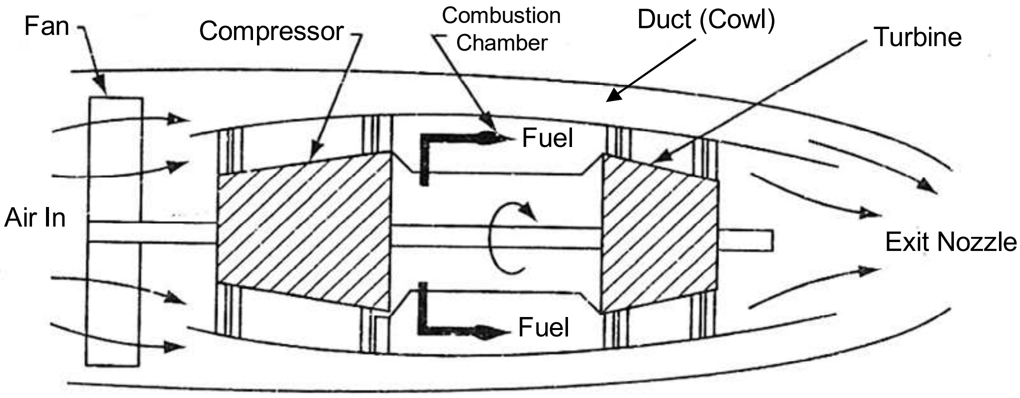


Fig. 1.23 Schematic diagram of a turbofan engine

- Fig. 1.23 shows a schematic diagram of a turbofan engine.
- It is a modification of turbojet engine which is most widely used in aircraft propulsion.
- In this engine, a large fan driven by the turbine forces a considerable amount of air through a duct (cowl) to the exhaust unit, bypassing the engine.
- A portion of air is sent to the engine compressor adding an advantage of creating supercharging effect.
- The fan exhaust leaves the duct at a higher velocity, enhancing the total thrust of the engine significantly.
- In this engine, the high-speed exhaust gases are mixed with the lower-speed air, which results in a considerable reduction in noise.

1.29 Ramjet engine (Lorin tube)

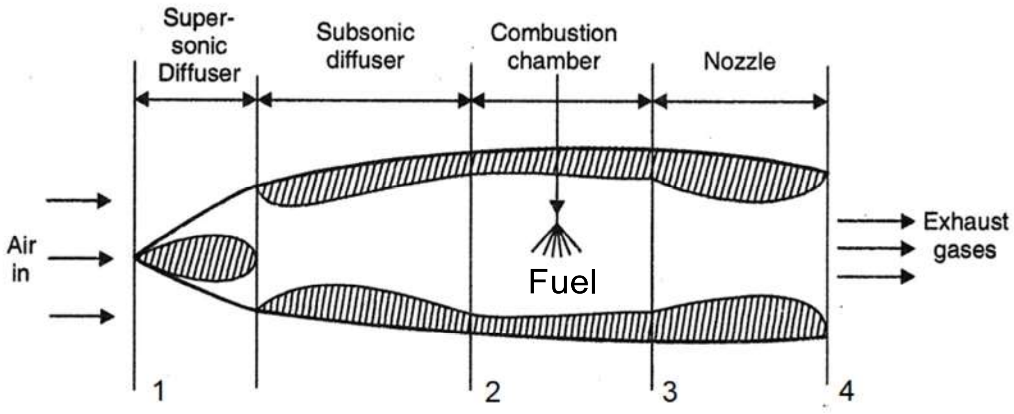
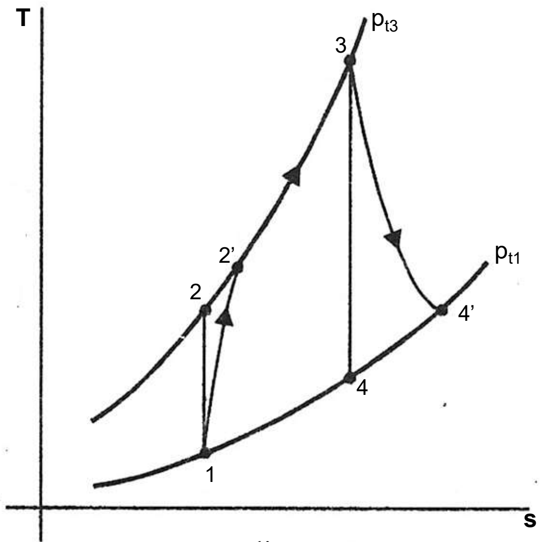


Fig. 1.24 Schematic diagram of a ramjet engine

- A ramjet engine is schematically shown in Fig. 1.24.
- A ramjet engine is a properly shaped duct with no compressor or turbine.
- It consists of a convergent-divergent diffuser, combustion chamber and an exit nozzle (or tail pipe).
- The supersonic and subsonic diffusers increases the pressure of inlet air to the ram pressure as it leaves the diffuser sections.
- The high pressure air is then heated to about 1500 K to 2000 K in the combustion chamber by injecting the fuel.



T-s diagram

- The high pressure and high temperature gases are then expand in the nozzle section creating very high velocity.
- These gases leave the engine with a very high speed and providing very high thrust.

1.29.1 Advantages and disadvantages of ram-jet engine

Advantages:

- Very simple and has no moving parts. Light in weight and maintenance-free.
- Any type of liquid fuel can be used.
- High temperature of the gases causes no danger since there is no turbine.
- Specific fuel consumption is lower than any other jet engines, in particular, at high speed and high altitude.

Disadvantages:

- Take-off thrust is zero, as air compression is achieved by ramming action. A small turbo-jet is required for starting the Ram-jet.
- The design of diffuser is very critical.
- Special flam stabilization techniques are needed in the combustion chamber to handle very high speed air.
- Due to very high temperatures, there is danger of dissociation of combustion products in the combustion chamber.
- Preferred only for high speed military aircrafts and missiles.

1.30 Pulse-jet engine

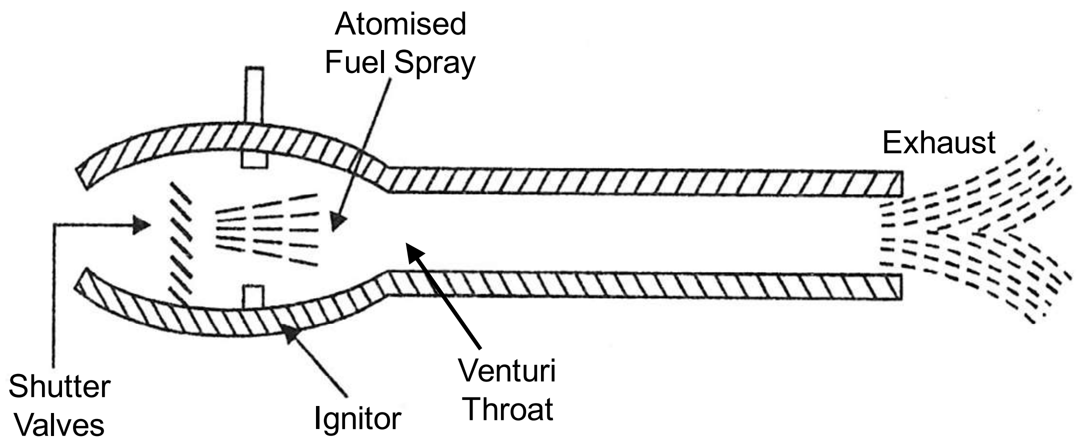


Fig. 1.25 Schematic diagram of a pulse-jet engine

- A pulse-jet engine is schematically shown in Fig. 1.25.
- Pulse-jet resembles a ram-jet except that it has shutter valves which make the process of air induction intermittent.
- The pulse-jet was developed in Germany during the World War II and its operating cycle may be compared with Otto cycle and is self-starting.
- The incoming air is compressed in the diffuser and the high pressure air enters the combustion chamber through the shutter valves.
- The fuel injected in the combustion chamber is burnt by the ignitor (or spark plug) and consequent rise in pressure closes the shutter valves, making the combustion process to occur at constant volume.
- The high pressure and high temperature gases expand through the nozzle producing the required thrust.
- After the gases leave the combustion chamber, the pressure decreases and the shutter valves open to allow the compressed air again. The air mixes with fuel and cycle repeats.

- Once the engine starts operating normally, the ignitor (or spark plug) is switched off and the residual flame inside the combustion chamber from the previous explosion is used for ignition in the succeeding cycles.
- Pulse-jet engines are much cheaper than Ram-jet engines and are self-starting. But, the propulsive efficiency is lower.

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