Module -1

Direct Current (DC) Circuits

CONTENTS:

D C circuits: Ohm's Law and Kirchhoff's Laws, analysis of series, parallel and series- parallel circuits excited by independent voltage sources. Power and Energy. Illustrative examples

Course Outcomes:-

After completing this chapter, students should be able to

- Understand the basic concepts in dc circuits.
- Solve for unknown quantities like resistance, current, voltage and power in series, parallel, and series - parallel circuits.
- Examine the applications of circuits.

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Introduction:

Electrical Energy: Every action in the Universe requires one or more forms of energy. Electricity is an exceptional type of energy. It can be static electricity or current electricity. Current electricity is dynamic and is defined as the flow of electrons in a closed path.

The word current derived from Latin word "currere" (meaning run, flow, move) has resulted in the word "current" in Middle English (English language spoken from circa 1150 to circa 1470). Current (charges)can easily be moved to any point along a couple of wires, often called interconnecting wires**.**

Current electricity is used for heating, lighting, chemical, magnetic and mechanical effects. Current Electricity or Electrical energy is a converted form of energy derived from that available in nature like fossil (remnant) [like oil, coal and natural gas], hydro, nuclear, wind, solar, Sea (Tidal, Wave and Ocean Thermal Energy),Geothermal, Biomass and gas, etc.

Conversion of energy from fossil, hydro, nuclear, wind, geothermal, tidal, wave to electrical energy is based on Faraday's laws of electromagnetic induction. Here the mechanical energy, derived from the above mentioned energy sources is converted into alternating current electrical energy.

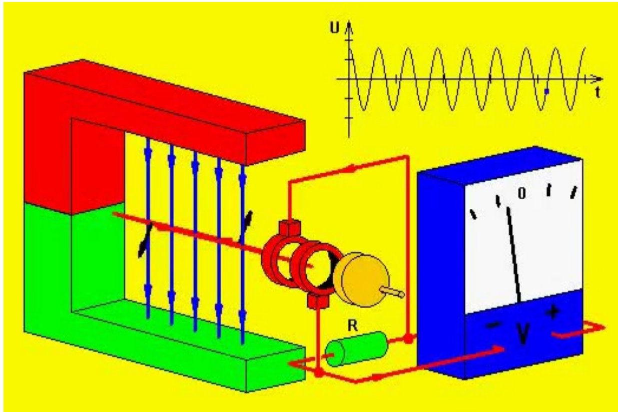
According to the law, whenever there is a relative motion between the flux and the conductor/coil/loop(turn)/circuit, an e.m.f is induced. This in turn drives a current through a closed path to supply electrical energy.

The e.m.f induced in a circuit is proportional to the time rate of change of the magnetic flux linking that circuit.

The flux can be from permanent or electro magnets.

A change in magnetic flux linking a circuit can be due to a change in the

- Magnetic field strength.
- Direction magnetic field.
- Position of the circuit.
- Shape of the circuit.
- Orientation of the circuit.
- In practice, large scale power in the range of hundreds of megawatt (MW), is generated by rotating coil/coils (driven prime movers like hydraulic, steam and gas turbines) in a uniform magnetic field of constant strength.
- The type of e.m.f induced is alternating of sine waveform (Refer Fig.1).
- When the e.m.f induced is alternating, the current is also alternating. As current refers to flow of charges, direction of flow of charges reverses periodically in case of alternating current (AC or ac).



In practice, alternating current power is delivered to industries, houses, office buildings, etc., because it is relatively easy to generate and transport over long distances.

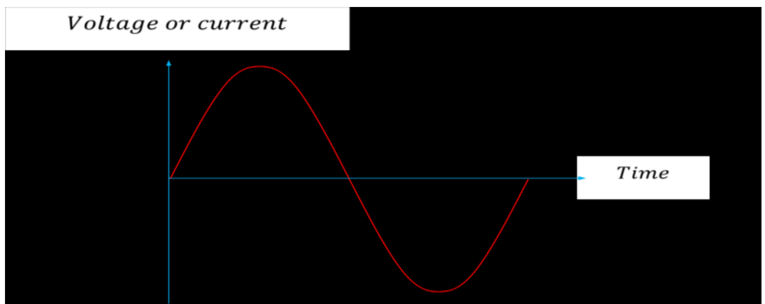


Fig. 2 Alternating voltage or current (Sine wave)

Not all the applications of electricity is based on ac. There are applications which calls for another type electrical energy, namely direct current (DC or dc) energy.

Direct current is the unidirectional current where the charge flow is unidirectional.

The voltage and current may remain constant or vary over time without changing the direction of charge flow (Fig.2). In other words polarity of dc source remains unchanged in either case.

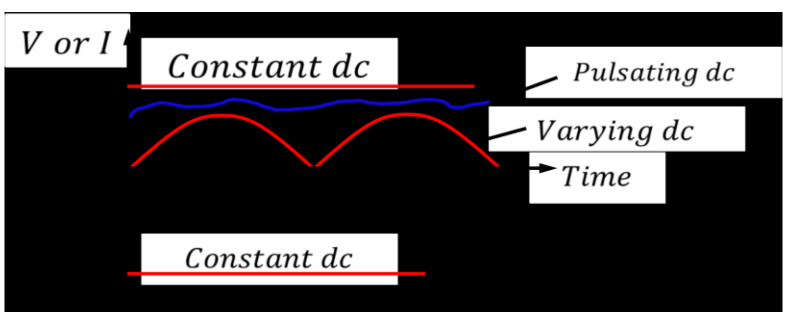


Fig. 2 Direct voltage or current

Batteries, solar cells, thermocouples and fuel cells are the source of dc power. It can also be obtained from a dc generator where the generated ac is converted into dc by commutator and brush assembly or by semiconductor rectifiers. For electrochemical processes, charging of battery, operation of electronic circuits etc., dc is essential.

Electric circuit:

Flow of charges or current requires a closed path called a circuit.

- An electric circuit is essentially a conduit that facilitates the transfer of charge from one point to another.
- The charges in motion transfer energy from the point of origin (say a battery) to utilization (say a lamp).
- Electric circuits, simple or complex, can be described in a variety of ways. It can be described through verbal description. As it does not provide a quicker mental picture of the actual circuit, it is seldom used.

The other ways of describing the circuits are,

- Block diagrams:

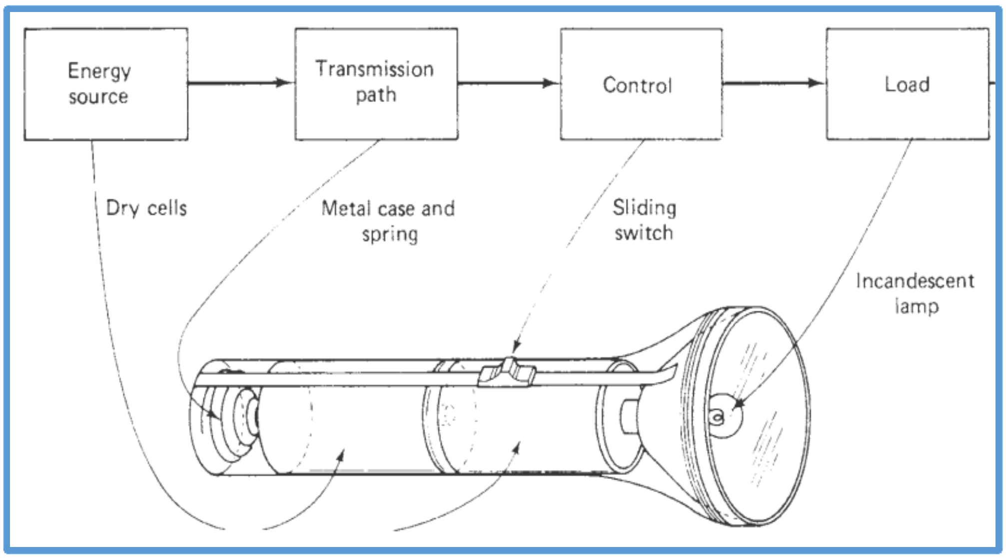


Fig.3 Cutaway view and block diagram of flashlight

Pictorials diagrams: Pictorial diagrams show the components as they appear actually in practice. Though they help to visualize circuits, they are cumbersome to draw. The figure.4 shows the pictorial view of a flashlight.

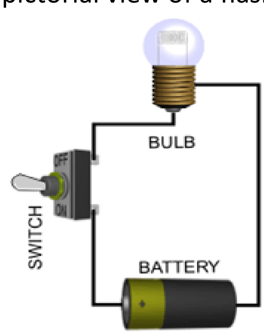


Fig.4Pictorial diagram of flashlight

Schematic diagrams: Schematic (meaning representational) diagrams use standard symbols to represent components. The symbolic components are normally connected by horizontal and vertical lines that are at right angles (Figure.5).

As resistance of the lamp filament is responsible in converting electrical energy to light, lamp is generally represented in terms of a resistance.

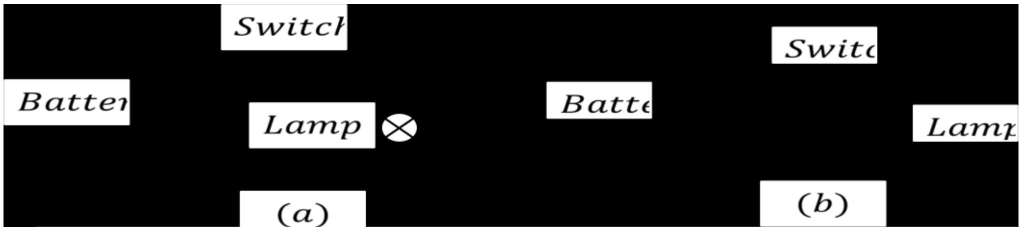


Fig.5 Schematic diagram of flashlight using (a)Lamp symbol (b)Resistance symbol

BASIC ELECTRICAL QUANTITIES: The three basic electrical quantities are Voltage, Current and Resistance.

VOLTAGE:

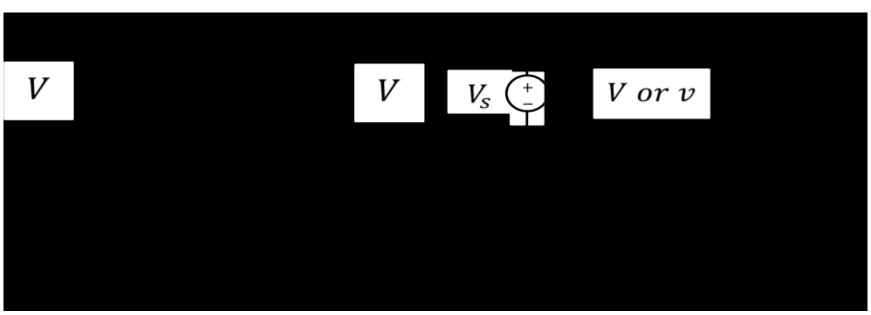
Voltage refers to an energy source (e.g. a battery) and is its ability to produce current within a closed path consisting of electrical components and provide energy to a load or loads. An electrical load consumes electric power to produce some form of work. For example, lights and electrical appliances are loads**.**

Definition of Voltage: Voltage is the work done per unit charge. Mathematically,

Voltage = work done in joule/charge in coulomb = (W/q) J/C

The unit of voltage is called volt and is indicated by letter v or V. While v is the time varying, V is time invariant**.**

The voltage is described by magnitude and direction. The voltage magnitude may be in µV, mV or kV and can be positive or negative. Conventionally, source terminals are marked plus and minus. As this is superfluous, one of them is omitted; generally minus mark is omitted. Another way f representing the source polarity is by an arrow with the head representing the plus sign and tail negative sign (figure. 6.)



Voltage is also called electromotive force(e.m.f), or potential and seldom pressure or tension while dealing with circuits. As the voltage is defined between points, voltage is also called potential difference.

In order to represent the voltage between two designated points, double script notation can be used. According to it, $V_{AB} = (V_A - V_B)$ indicates that point A is positive with respect to point B or A is at higher potential with respect to the potential of B. obviously, , $V_{BA} = -V_{AB}$.

CURRENT

Under the influence of electric force or voltage, the charges(electrons) move in a circuit. Moving electrons is recognizes by the term electric current. The symbol i or I used for current. While i indicates a time dependent current, I represents a time independent current. The intensity (magnitude) of current is measured by the amount of charges that flows past a given point at a certain time.

If Q is the charge in coulombs moving past a point at a time t, then Current = Charge / time or I = Q/t, coulombs/ second. Alternately it is the time rate of charge transference between two points in a circuit. i.e., i = dq/dt.

The current is described by the magnitude and direction. The unit of current is ampere and the symbol is A. Often current is indicated by amp or Amp, but both of them are informal and unofficial.

The current magnitude, in the ascending order may be in nA, µA, mA or kA. The direction of current flow is indicated by an arrow. The head of the arrow indicates the direction of current flow. The direction of current can be indicated through double subscript notation also. For example, I_{AB} indicates that the current flow is from A to B, where A is at higher potential with respect to point B. Obviously $I_{AB} = -I_{BA.}$

The current

(a) In a circuit always flows from the point of more positive potential to the point of greatest negative potential.

(b) Always takes a least resistance path.

RESISTANCE :

The resistance is a measure of resistor's opposition to the flow of current. A resistor is a component of a circuit. The symbol used for resistance is r or R and the circuit symbol is shown in figure.7. The unit of resistance is ohm and the abbreviation for ohm is the Greek letter omega Ω.

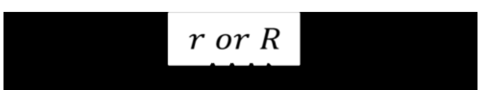


Fig.7 Circuit symbol for Resistance

At any specified temperature, the resistance $R = \frac{\rho l}{a}$, where ρ (rho) is the resistivity or volume resistivity or specific resistance of the material. The unit of specific resistance is ohm –cm when resistance is in ohm, length in centimeter and cross sectional area in sq. cm.

Often, the resistivity is specified in $\Omega/m/mm^2$. Resistivity is a measure of how strongly a material opposes the flow of electric current. A low resistivity indicates a material that readily allows the movement of electric charge.

Though the expression $R = \frac{\rho l}{a}$ does not include a term involving temperature, it is a function of temperature. While an increase in temperature increases the resistance of a metal conductor, a decrease in temperature decreases the resistance of the conductor. Knowing the resistance and temperature coefficient at temperature t_1, the resistance at temperature t_2 can be calculated by the following formula:

$$R_2 = R_1[1 + \alpha_1(t_2 - t_1)]$$

The reciprocal of resistance is conductance and is abbreviated by the letter G. The unit of conductance is mho or Siemens.

The reciprocal of resistivity is called the conductivity and is represented by the Greek letter sigma,σ. Mathematically, $\sigma = 1/\rho$ and the unit is mho/cm or S/cm,

Resistors are introduced intentionally in circuits to convert electrical energy into heat and light, to function as a voltage divider, current limiter etc

Ohm's Law:

Ohm's law precisely expresses the relationship between current, voltage and resistance

Statement of the law:

"At constant temperature or the resistance remaining constant, the current is directly proportional to the emf impressed to a circuit and inversely proportional to the resistance of the circuit".

$I = \frac{V}{R} = GV, V = IR \; or \; R = \frac{V}{I}$ where V = voltage applied tot eh conductor or circuit.

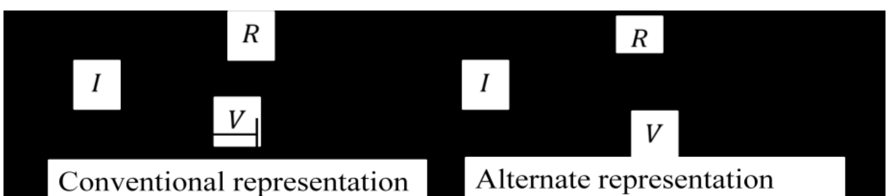


Fig.1 Circuit having a voltage source and resistance

Work (energy) and power

Energy:

Energy refers to ability or capacity to do work.

- Work is the transferring or transforming of energy.
- Therefore, Work is done each time energy changes from one form to another.
- A load, which receives electrical energy from an

energy source (battery and generator), does work to convert it into different forms.

The loads, for example, are

(a) Electric Lamp: converts electrical energy into light energy.

(b) Electric Stove: converts electrical energy into heat energy.

(c) Electric Motor: converts electrical energy into mechanical energy

(d) Electric Fan: converts electrical energy into wind energy

(e) Speaker: converts electrical energy into sound energy.

(f) Electrical energy is converted to chemical energy by electrolysis

As a general rule, load consumes major portion of the energy supplied by the source. Since, voltage V is defined as the work done per unit charge and charge Q = It,

Energy or work W = V x Q = V x It = VIt.

The unit of energy is Joule and is symbolically represented by the letter J.

Since V = IR, according to Ohm's law, $W = VIt = I^2Rt = V^2t/R$ Joules

POWER

Power refers to the rapidity of energy conversion or consumption. In other words, Power is a measure of the rate at which work is done or at which energy is converted from one form to another. For example, a flashlight expends a given amount of energy much faster than an electronic watch. In other words, flashlight requires more power to function than an electronic watch. Thus, power is defined as the rate of doing work.

Therefore, $Power = \frac{Work\ done}{time} = \frac{VIt\ or\ I^2RT\ or \frac{V^2}{R}t}{t} = VI\ or\ I^2R\ or \frac{V^2}{R}$

The unit of power is watt (or joules per second) and is symbolically represented by the letter W. When the power is large, it is generally expressed in kilowatt (kW) or megawatt (MW).

In any circuit, power supplied by the source or sources must always be equal to the power dissipated.

In practice all the components in the dc circuit including connecting wires consume power. While the power dissipated in the load is useful, the power dissipated in other components is considered as loss.

Every practical device has a limitation on the amount of power that it can handle or the temperature rise it can withstand, beyond which it gets damaged. Therefore, all devices are rated in terms of power and voltage. The devices perform efficiently at the rated power.

For example, resistors must be operated within specified temperature limits to avoid permanent damage to it. The temperature limit is defined in terms of the maximum power, called the power rating

The maximum voltage that may be applied to the resistor is called the voltage rating and is related to the power rating by $V = \sqrt{PR}$, where V is the voltage rating in volts, P is the power rating in watts and R is the resistance in ohm.

Similarly, the maximum current that can be allowed to pass through a resistor is $I = \sqrt{P/R}$

PRACTICAL UNIT OF ENERGY

Joule (J) which is the same as watt – second is the unit of energy and is rather a small unit. For this reason, it is not suitable for energy billing. For example, the energy required to raise the temperature of one litre of eater is to boil from 15 C is 355410.5 J. To overcome the use such a large number, the electric power companies measure the energy consumption in kilowatt-hour (kWh).

The kilowatt hour is defined as the energy dissipated in a 1kW load in one hour. Therefore, 1kWh = 1000 x 3600 = 3.6 x 10^6 joule or watt second.

Thus, the energy required to boil water, mentioned above , can be expressed as $\frac{355410.5}{3.6\ x10^6} = 0.1\ kWh.$

A kWh is referred to as one unit in practice. The generation of energy of a power plant is measured, generally, in millions of units. The energy is measured by energy meters or watt-hour meters.

INDEPENDENT AND DEPENDENT VOLTAGE SOURCES

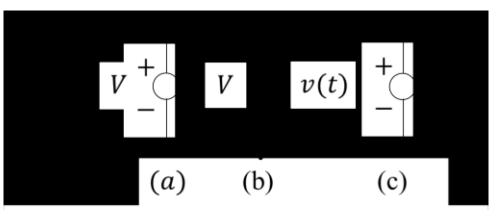
An independent voltage source is one whose output voltage (dc or ac) is unaffected by any other quantity.

A dependent voltage source is one, whose voltage varies with other variables (voltage or current) somewhere else in the circuit.

Further, independent voltage sources can be classified as ideal and practical (real) sources

INDEPENDENTVOLTAGE SOURCES

An ideal voltage source is characterized by its voltage which remains constant irrespective of the load connected across its terminals.



$Fig. 2\ Circuit\ symbol\ for\ independent\ ideal\ oltage\ source\ (a) dc\ generator\ (b)\ battery\ (c)\ ac$

.

It supplies any amount of current at constant voltage. The figure.2 shows the ideal voltage sources. An ideal voltage source is assumed have no internal loss of energy and hence no internal resistance. Hence the voltage at the terminals remains the same irrespective of the current supplied by it.

However, every practical voltage source will have some internal loss and therefore will not maintain a constant voltage at all loads. This effect is considered in terms of internal resistance or source resistance.

For analysis purposes, the internal resistance is assumed to be in series with the voltage source (Fig.3).

Because of internal resistance, the terminal voltage of a practical voltage source does not remain constant with load changes. Unless otherwise stated, the independent voltagesources are assumed as ideal.

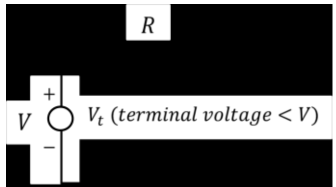


Fig. 3 *Practical independent voltage source*

Energy Sources can provide constant voltage so long they are powered with an input.

For example, a continuously powered generator can supply power maintaining constant voltage. On the other hand, a battery cannot maintain a constant voltage over time (meaning that the voltage will drop as the battery is used) at its terminals because it loses its stored energy or lose its charge. However for most purposes the voltage can assume to remain constant.

SUPPLY VOLTAGE, APPLIED VOLTAGE or OPEN CIRCUIT VOLATGE

The voltage across a source (e.g. battery) when not connected to an external circuit (i.e., when the current driven by the source is zero) is called the Electromotive force (emf), supply voltage, applied voltage, source voltage or open circuit voltage.

In an ideal case, this is the voltage applied to a circuit connected to the source

LOAD VOLTAGE, VOLTAGE RISE AND VOLTAGE DROP:

Load Voltage: Voltage across the load is called the load voltage.

Voltage drop: Voltage across a component in a circuit is often called voltage drop when current flows from a higher potential point to a lower potential point in the circuit.

Voltage rise: Voltage across a component is called the voltage rise when current flows from a lower to a higher potential point in the circuit.

ASSUMPTION IN SOLVING PROBLEMS

The connecting wires are assumed to have zero resistance.

- Therefore, the voltage drop in them is zero. In other words, all points on the connecting wire are at same potential.
- Also, there will be no power loss in the connecting wires.

However, in practice all these ideal situations are not met.

Circuit (network) analysis

Circuit (network) analysis is a mathematical process and its goal is to determine the voltages across, and current through, every component in the circuit planned for a specific purpose.

As the circuit becomes more complex (with resistances connected in different configurations and not possible to reduce all of them to a single resistance), it is not possible to find V and I corresponding to different components in the circuit easily by Ohm's law. In this regard Kirchhoff's laws help to solve the problem

KIRCHHOFF'S LAWS

There are two Kirchhoff's laws; the current law and voltage law. These laws arise from conservation laws

Kirchhoff ˊs First law or Kirchhoff ˊs Current Law (KCL):

The algebraic sum of electric currents at any junction (node) of an electric circuit is equal to zero at every instant of time.

Node: A node is a junction (connection point) at which two or more elements are connected. In figure.4, point O is a node.

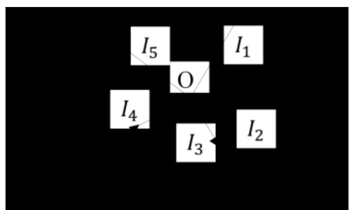


Fig.4 Kirchhoff's current law

If the direction of current, entering and leaving a node are considered to be positive and negative respectively, then with reference to fig.4

$$-I_1 + I_2 - I_3 + I_4 + I_5 = 0$$

Or $\sum_{n=1}^{N} I_n = 0$

KIRCHHOFF ˊS SECOND LAW OR KIRCHHOFF ˊS VOLTAGE LAW (KVL):

"The algebraic sum of the voltages around a closed loop is equal to zero at every instant of time"

OR

"The algebraic sum of all the voltage rises must be equal to the algebraic sum of all the voltage drops at every instant of time"

CLOSED LOOP

A closed-loop circuit or a loop is a conducting path in a circuit that has the same starting and ending points. If the current flowing through a circuit returns to the same starting point from any point, it would be a closed-loop circuit

While tracing through the loop, voltage encountered may be voltage of the sources or voltage drops across the resistances. While writing the KVL equations, the following sign convention can be adopted.

SIGN CONVENTION FOR VOLTAGE

(a) If the voltage of the source is traced from negative to positive terminal, then the voltage of the source can be taken with a positive sign. Otherwise with a negative sign.

(b)The voltage drop across a resistance when traced in the direction of current can be taken with a negative sign. Otherwise with a positive sign.

The sign conventions need not be the same; it can be opposite to what has been stated under (a) and (b).

(c) The choice of starting point while traversing through the circuit is entirely arbitrary, and the path traversed may be in clockwise or counter clockwise sense.

KIRCHHOFF ´S VOLTAGE LAW (KVL)

Application of KVL to the circuit shown in fig. 5 leads to,

$$E_1 - V_1 + E_2 - V_2 - V_3 = 0$$

OR

$$E_1 + E_2 = IR_1 + IR_2 + IR_3$$

In general, $\sum V_n = \sum IR$ or $\sum_{n=1}^{N} V_n = 0$

Where N is the number of circuit elements in the closed path of the circuit.

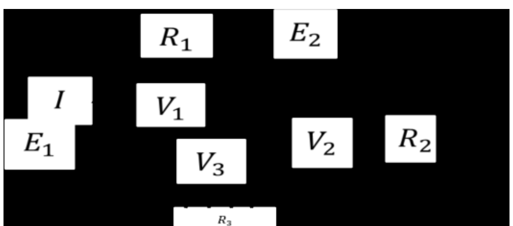


Fig. 5 Kirchhoff's voltage law

RESISTANCES IN DIFFERENT COMBINATIONS

The resistances can be connected in series, parallel, series – parallel etc. Each configuration has its own applications.

RESISTANCES IN SERIES

Resistances are said to be in series when two are more resistors are connected end to end in a circuit so that the same current flowing in one resistor also flows through the others. The voltage across each resistor depends on the value of the resistance.

RESISTANCES IN SERIES:

If V_1 and V_2 are the voltage drops across R_1 and R_2, then from Ohm's law $V_1 = IR_1$ and $V_2 = IR_2$, where I is the current flowing through the circuit.

From Kirchhoff's voltage law, $V = V_1 + V_2 = IR_1 + IR_2 = I(R_1+R_2) = I\,R_{12}$

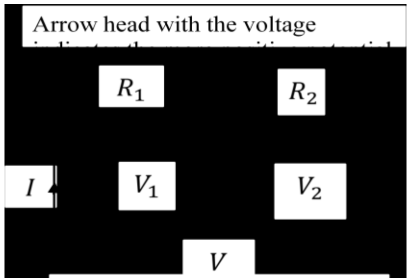


Fig.6 Resistances in series

Where R_{12} is the equivalent resistance of R_1 and R_2 in series. It is also the total resistance of the circuit as seen by the source. In general, the total or equivalent resistance of a series circuit is the sum of individual resistance. That is, R_{eq} or $R_{123n} = R_1 + R_2 + R_3 + \ldots\ldots + R_n$ and is the ration of source voltage to circuit current.

i.e., $R_{12} = V/I$

It clear, that KVL helps to replace all the resistors with an equivalent resistance and offers a general rule to find the equivalent resistance of a series circuit.

The equivalent resistance of a series circuit is the sum of all resistances and will always be larger than the largest resistance in the circuit.

Power consumed in R1 and R2 are respectively

$$P_1 = I^2 R_1 = V_1 I = \frac{V_1^2}{R_1}$$

and

$$P_2 = I^2 R_2 = V_2 I = \frac{V_2^2}{R_2}$$

Total power consumed, P = $P_1 + P_2 = I^2 R_1 + I^2 R_2 = I^2 (R_1 + R_2) = I^2 R_{12}$ watts

$$= V_1 I + V_2 I = (V_1 + V_2) I = VI\ watts$$

$$= \frac{V_1^2}{R_1} + \frac{V_2^2}{R_2} = \frac{(IR_1)^2}{R_1} + \frac{(IR_2)^2}{R_2} = \frac{V^2 X R_1^2}{(R_1+R_2)^2 x R_1} + \frac{V^2 X R_2^2}{(R_1+R_2)^2 x R_2} = \frac{V^2}{(R_1+R_2)} = \frac{V^2}{R_{12}} \text{ watts}$$

Increase in voltage increases the current in the circuit and the power consumed by each resistor. If it is beyond the power rating of the resistor, the heat developed in the resistor will destroy it.

APPLICATIONS OF SERIES CIRCUIT:

Series topology can be used as a voltage divider, to limit the current in a circuit, etc.

A voltage divider can be fixed voltage divider or variable voltage divider.

Fixed voltage dividers are used for biasing of transistors (figure.7), measurement of high voltage etc.

A variable voltage divider (generally called potentiometer or pot), for example, is used to apply variable voltage to power amplifiers to control its output or input to a speaker. This in turn controls the audio output of the speaker (Fig.8).

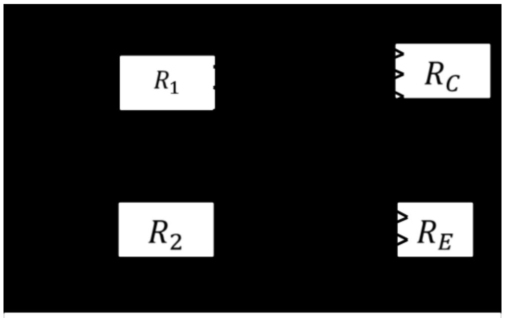


Fig. 7 Biasing of Transistor R_1 and R_2 forms the voltage divider

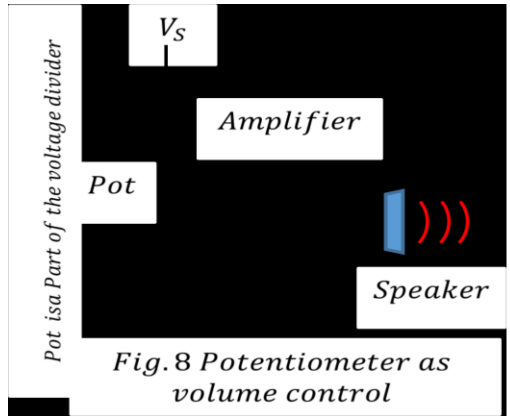


Fig. 8 Potentiometer as volume control

RESISTANCES IN PARALLEL:

Resistances are said to be in parallel when two are more resistors are connected side by side in a circuit so that the voltage existing across one resistor also exists across the others and division of current occurs at the junction.

The current in each branch depends on the resistance of that branch.Figure.9 shows a parallel circuit

If I1 and I_2 are the currents in R_1 and R_2, then from Ohm's law

$I_1 = V/R_1$ and $I_2 = V / R_2$ where V is the applied voltage to the circuit

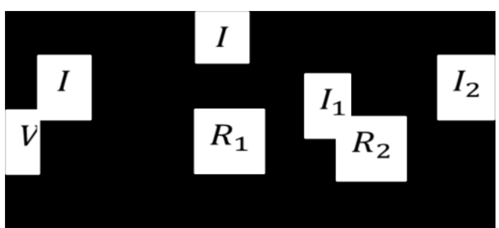


Fig. 9 Parallel circuit

Thus from Kirchhoff's current law**,**

$$\boldsymbol{I = I_1 + I_2 = \frac{V}{R_1} + \frac{V}{R_2} = V\left(\frac{1}{R_1} + \frac{1}{R_2}\right) = V\left(\frac{R_1 + R_2}{R_1 R_2}\right) = \frac{V}{R_{12}}}$$

Where $R_{12} = \frac{R_1 R_2}{(R_1 + R_2)}$ is the equivalent resistance of R_1 and R_2 in parallel. It is also the total resistance of the circuit.

In general, the total resistance of a parallel circuit is

$$R_{eq} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots \ldots + \frac{1}{R_n}}$$

The total resistance of the parallel circuit will be always less than the least resistance of the resistors in parallel. It is the ratio of the voltage across the parallel circuit to the current supplied by the source.

Therefore $R_{12} = V / I$

Power consumed in R1 and R2 are respectively

$$P_1 = I_1^2 R_1 = VI_1 = \frac{V^2}{R_1}$$

$$P_2 = I_2^2 R_2 = VI_2 = \frac{V^2}{R_2}$$

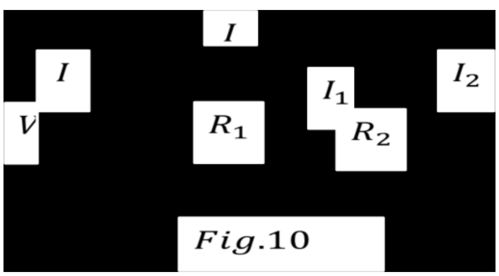
Total power consumed by the circuit or input to the circuit is $P = P_1 + P_2 = I^2 R_{12} = VI = \frac{V^2}{R_{12}}\ watts$

As the number of resistances connected across fixed source voltage (ideal source) increases, the current in the already existing resistors remains the same but current supplied by the source increases to meet the additional demand. In all power distribution network Parallel circuits are used

Current Divider Rule:

The current divider rule offers a shortcut approach to find individual branch currents in a parallel circuit without knowing the value of the applied voltage.

Consider two resistances R_1 and R_2 connected in parallel across a source of voltage V as shown in figure 10.



$Fig.10$

Current through the resistance R_1 is $I_1 = V / R_1$.

Since V = I R12 = I [(R1 x R2)/ (R1 + R2)],

$$\boldsymbol{I_1 = \frac{I\frac{R_1R_2}{(R_1+R_2)}}{R_1} = I\frac{R_2}{(R_1+R_2)}}$$

Similarly,

$$\boldsymbol{I_2 = \frac{I\frac{R_1R_2}{(R_1+R_2)}}{R_2} = I\frac{R_1}{(R_1+R_2)}}$$

From the foregoing, current divider theorem for two resistors in parallel can be stated in the equation form as

$$\boldsymbol{I_n = \frac{I\ x\ Resistance\ of\ the\ opposite\ branch, i.e., R_{opposite}}{Sumof\ the\ two\ resistances\ i.e., (R_n + R_{opposite})}}$$

Where In = current in branch n, Rn = resistance of branch n.

RESISTANCES IN SERIES – PARALLEL:

The circuits used in radio receivers, television sets and others are the combinations of series and parallel circuits with different types of components. Series - parallel circuits combine the characteristics of series and parallel circuits. Rules meant for combining the series and parallel resistances can be used to reduce the series parallel circuits to simple series or parallel equivalent circuits. The currents and voltages of the circuit can be found using Ohm's law, current divider rule and voltage divider rule. Figure 11 shows a series parallel circuit.

Since R1 is in series with the parallel combinati

of R2 and R3, total resistance across the supply

is $R_{eq} = R_1 + \frac{R_2 R_3}{(R_1+R_2)}$

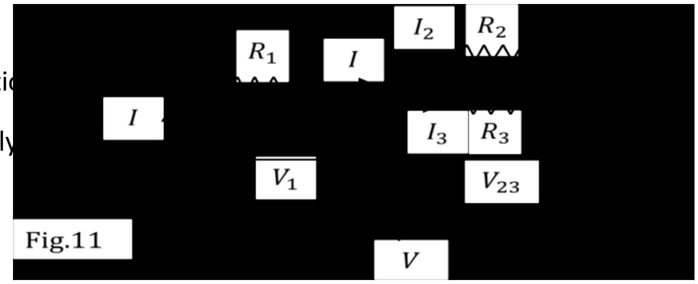


Fig.11

Current supplied by the supply, I = V /R_{eq}

Voltage drop across R1 is V1 = I R_1

Power consumed by R_1 is $P_1 = I^2 R_1 = V_1 I = V_1^2/R_1$

From current divider rule, $I_2 = I[R_3/(R_1 + R_2)]$

Similarly, $I_3 = I[R_2/(R_1 + R_2)]$ or $I_3 = I – I_2$

Voltage drop across R_3 or R_2 is $V_{23} = I_2 R_2 = I_3 R_3$

Power consumed by R_2 is $P_2 = I^2{}_2 R_2 = V_{23} I_2 + V^2{}_{23} /R_2$

Power consumed by R_3 is $P_3 = I^2{}_3 R_3 = V_{23} I_3 + V^2{}_{23} /R_3$

ANALYSIS OF CIRCUITS WITH MULTIPLE LOOPS AND HAVING ONE OR MORE VOLTAGE SOURCES:

With increased number of elements and structurally more complicated circuits, network reduction becomes cumbersome. In such cases, the network can be solved using Kirchhoff's laws. Among the different methods available,

Branch Current and Loop Current (also known as mesh current) methods are elementary.

Branch Current method:

In this method, assuming a current in each branch, simultaneous voltage and current equations are formulated using Kirchhoff's laws to find current in each branch of a circuit.

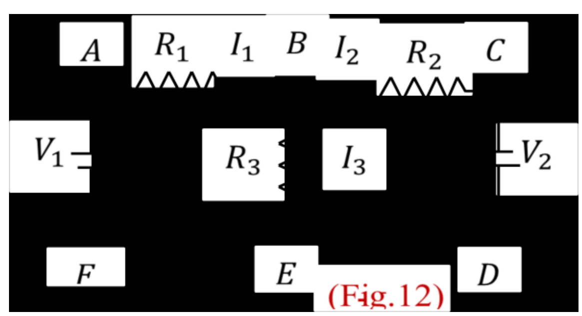
The KCL equations are $I_1 = I_1$ at node A

$I_2 = I_2$ at node C and

$I_3 = I_1 + I_2$ at node B ………. (1)

$V_1 - I_1 R_1 - I_3 R_3 = 0 \dots\dots\dots (2)$ for loop FABEF

$I_3 R_3 + I_2 R_2 + V_2 = 0 \dots\dots\dots. (3)$ for loop EBCDE



(Fig.12)

Solution of equations (1), (2) and (3) leads to the values of branch currents.

LOOP OR MESH ANALYSIS:

In mesh analysis, the assumed mesh currents circulate through all the elements in their respective meshes. Though the mesh currents can be assumed in any direction, it is customarily taken in a clockwise direction. There is no significant reason for this, except perhaps convention

The number of loops considered must be such that, all the elements in the circuit are covered by one or the other loop currents or KVL must be applied a sufficient number of times to include every element in the network at least once

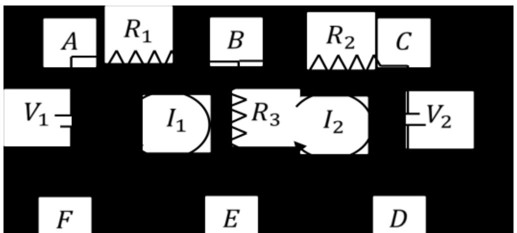
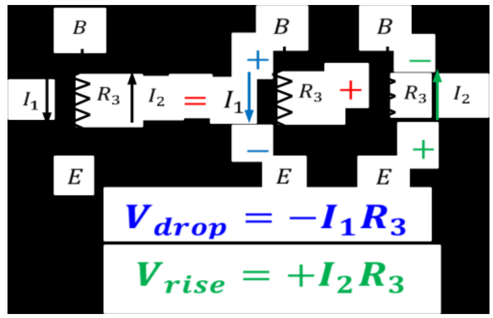


Fig. 13 Illustration of loop analysis



Application of KVL to loop FABEF leads to

$$V_1 - I_1R_1 - I_1R_3 + I_2R_3 = 0$$

$$-I_2R_3 + I_1(R_1 + R_3) = V_1 \quad \dots\dots\dots(1)$$

Application of KVL to loop BCDEB leads to

$$-I_2R_2 + V_2 - I_2R_3 + I_1R_3 = 0$$

$$-I_1R_3 + I_2(R_2 + R_3) = V_2 \quad \dots\dots\dots(2)$$

The solution of equations (1) and (2) leads to I_1 and I_2 and all the branch currents and voltages can be obtained.

ANALYSIS OF CIRCUITS WITH MULTIPLE LOOPS AND HAVING ONE OR MORE VOLTAGE SOURCES:

Simultaneous equations can be solved by:

(a) Substitution method

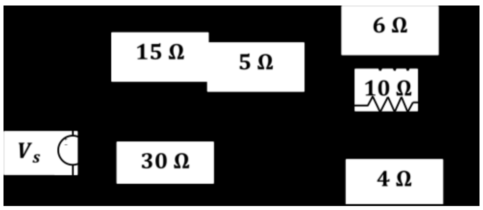
(b) Elimination method or

(c) Determinant method

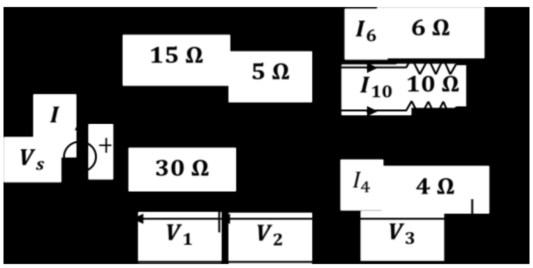
Problem: 1

[redacted]

Solution:



[redacted]

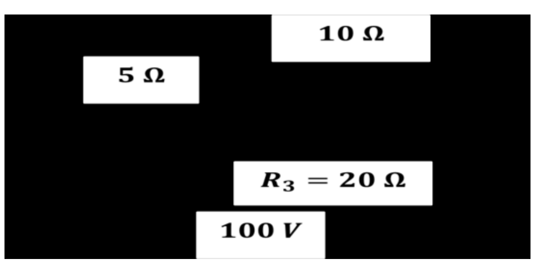


[redacted]

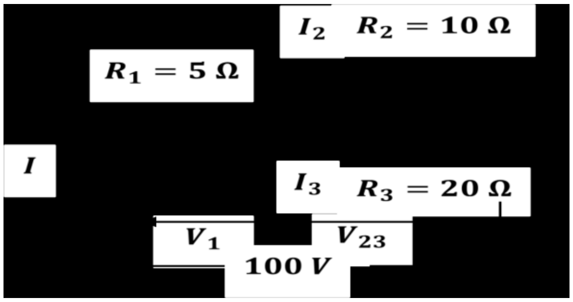
[redacted]

Problem 2:

[redacted]



Solution:



[redacted]

[redacted]

[redacted]

[redacted]

Alternately,

[redacted]

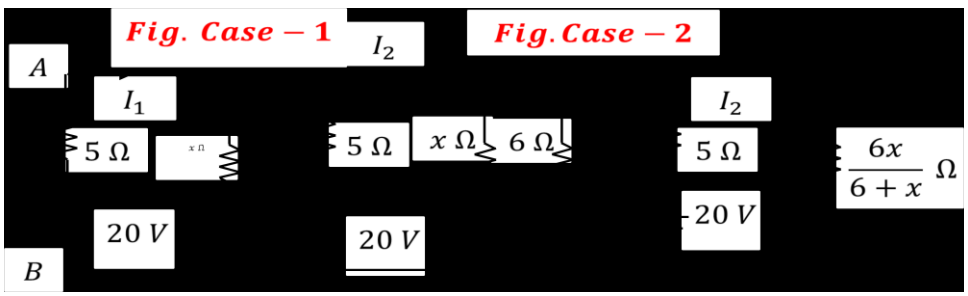
$367.2\ W$

[redacted]

[redacted]

Problem 3:

[redacted]



[redacted]

[redacted]

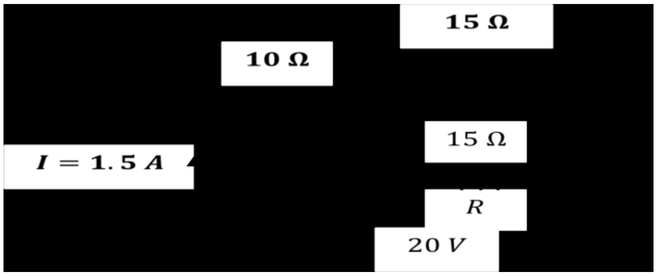
$$6x^2 - 25x - 150 = 0$$

[redacted]

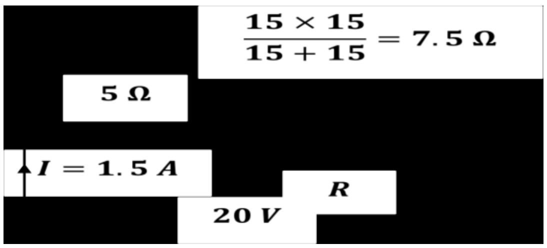
$$x = 7.5\ \Omega$$

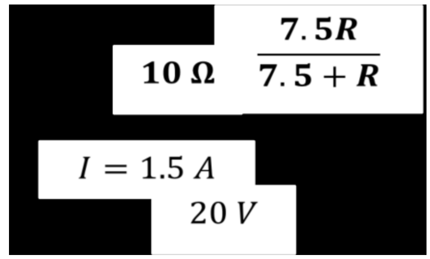
Problem 4:

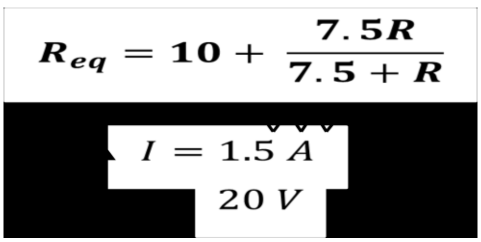
[redacted]



Solution:







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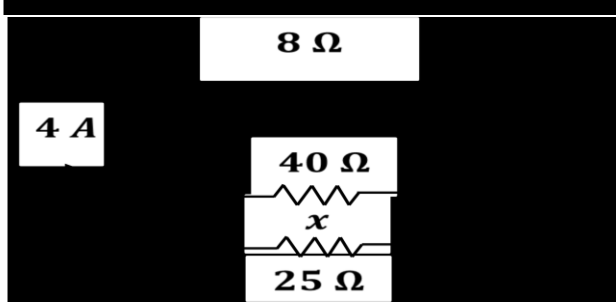
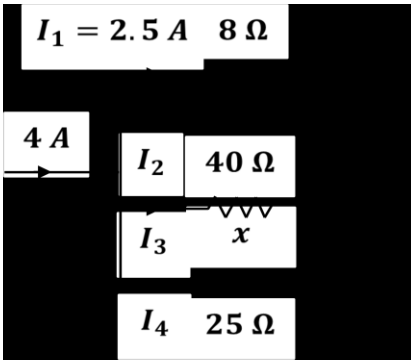
Problem 5:

[redacted]



Solution:

[redacted]



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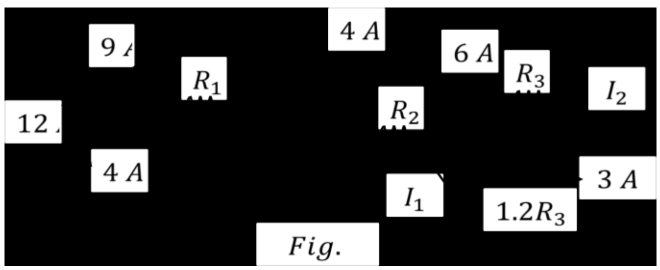
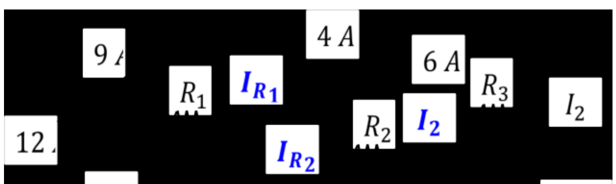
Problem 6:

[redacted]





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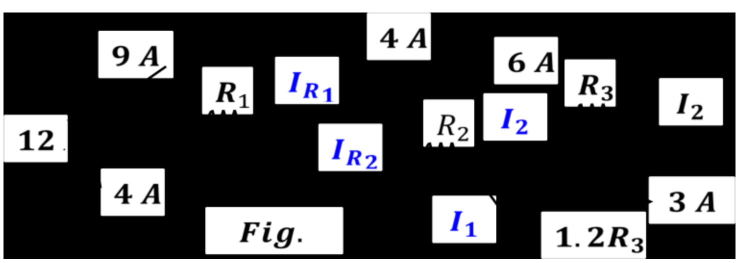
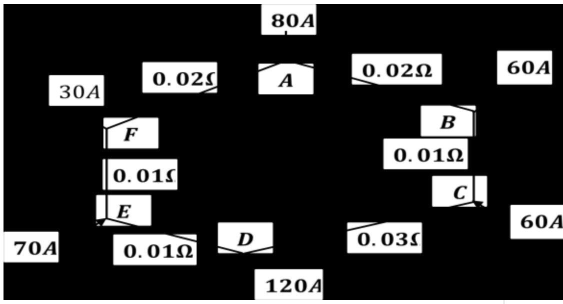
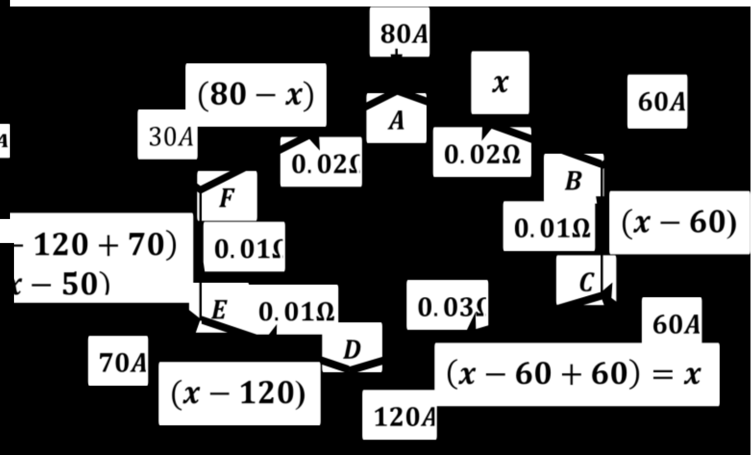


[redacted]

[redacted]

Problem 7: **Find the current in all branches of the network shown**

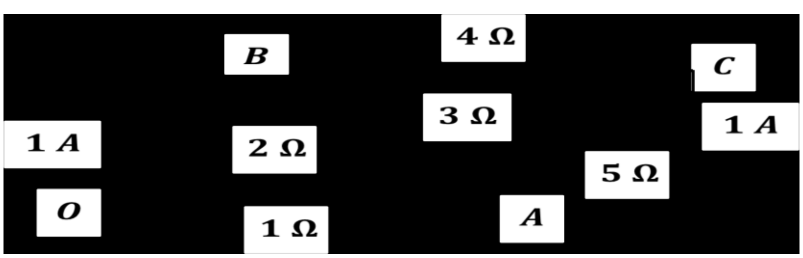




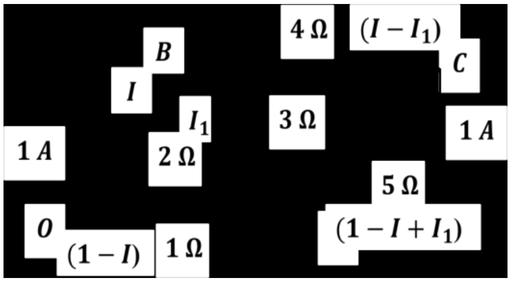
If a current of x is assumed to be flowing along AB, the current distribution, from KCL, is as shown in figure.

$x = 39\ A$

Problem 8: **Find the current distribution in the network shown.**



Solution:



[redacted]

[redacted]

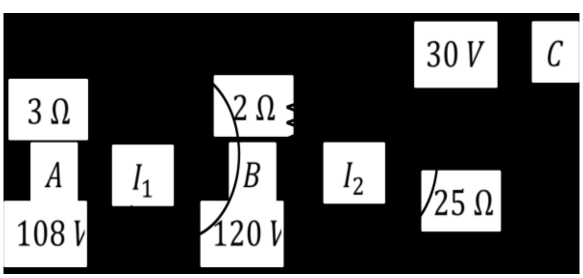
[redacted]

Problem 9: Two batteries A and B are joined in parallel. Connected across the battery terminals is a circuit consisting of a battery C in series with 25 Ω resistor, the negative terminal of C being connected to the positive terminals of A and B. Battery A has an emf of 108 V and an internal resistance of 3 Ω, and the corresponding values for battery B are 120 V and 2 Ω.

Solution:

Battery C has an emf of 30 V and a negligible internal resistance.

Determine (a) the value and the direction of the current in each battery and (b) the terminal voltage of battery A. State whether the batteries are discharging or getting charged.



[redacted]

Current through battery A is $I_1 = -0.183\ A$.

[redacted]

current through B is
$I_2 - I_1 = 5.54 - (-0.183) = 5.723\ A$

Current through battery C is $I_2 = 5.54\ A$

[redacted] $= 108.55\ V$

[redacted] B *and* C, they are discharging

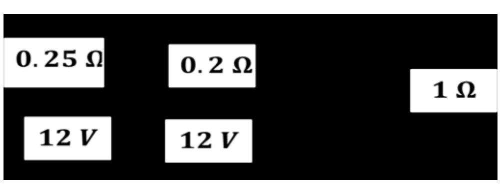
[redacted] battery A at its positive terminal, it is getting charged.

Check: [redacted]

Problem 10:

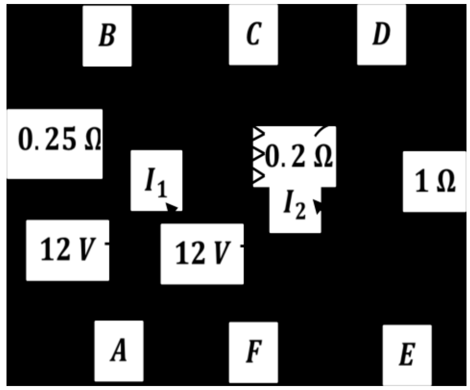
[redacted]

Solution:



Solution:

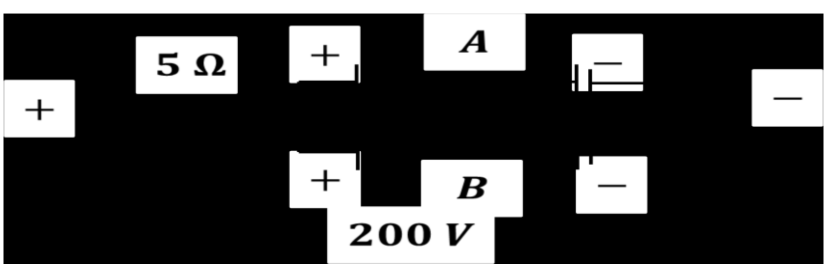
[redacted]

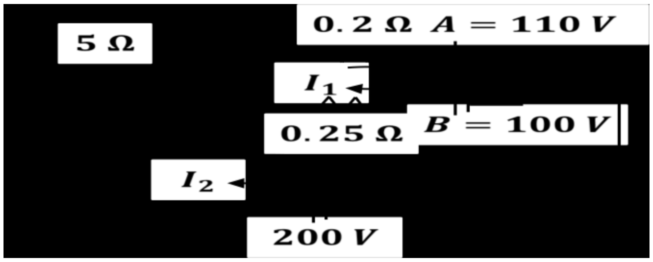


[redacted]

Problem 11:

[redacted]





[redacted] **(1)**

[redacted] **(2)**

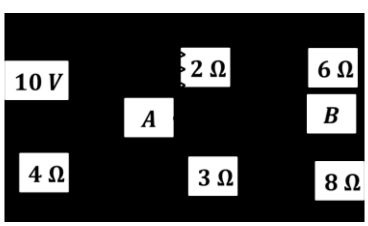
$I_1 = -11.96\ A$ $I_2 = 18.48\ A$

[redacted]

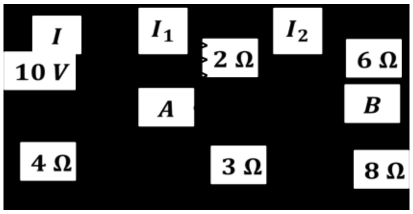
$I_2 = 18.48\ A$

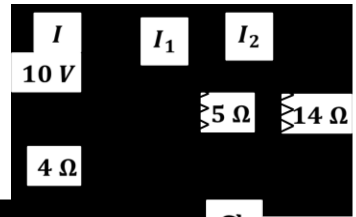
Problem 12:

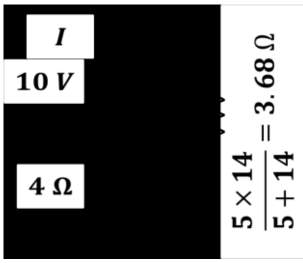
Find current in the battery, the current in each branch and potential difference across A and B in the network shown.

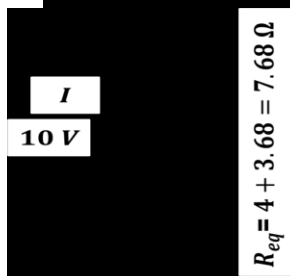


The currents can be obtained by using network reduction technique or by using KVL.









[redacted]

[redacted]

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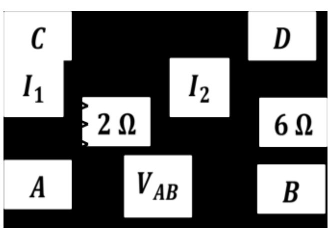

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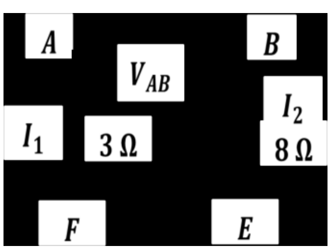
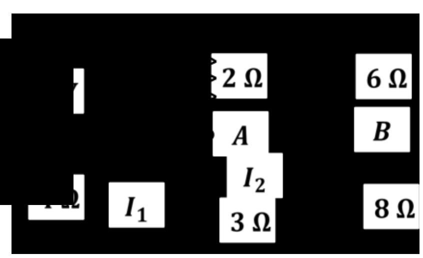


[redacted]

[redacted]



[redacted]

$I_1 = 1.3\ A\ \ and\ \ I_2 = 0.342\ A$

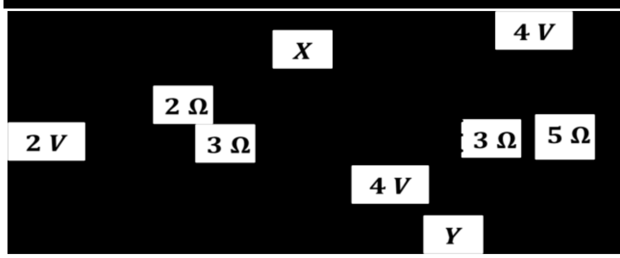
$I_1 = 1.3\ A$

$I_1 - I_2 = 1.3 - 0.342 = 0.958\ A$

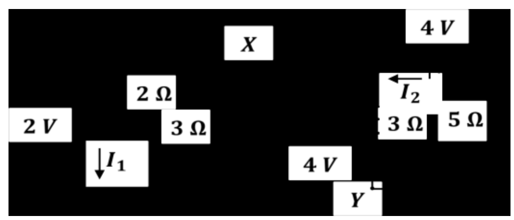
$I_2 = 0.342\ A$

Problem 13:

[redacted]



Solution:

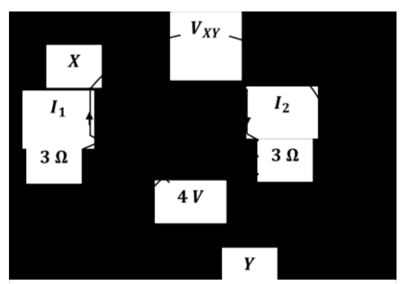


[redacted]

$I_1 = \frac{2}{2+3} = 0.4\,A$

$I_2 = \frac{4}{3+5} = 0.5\,A$

[redacted]

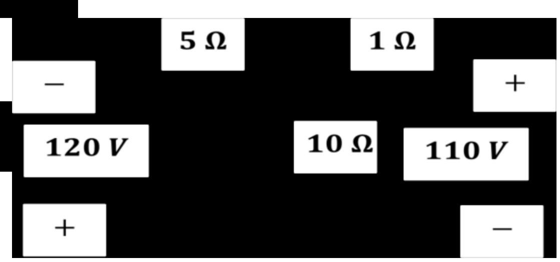


Point X is considered positive

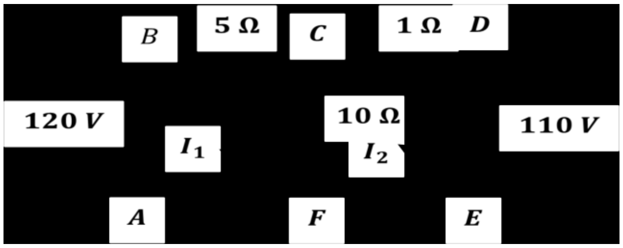
[redacted]

Problem 14

[redacted]



Solution:



[redacted]

[redacted]

$-15I_1 + 10I_2 = 120 \quad (1)$

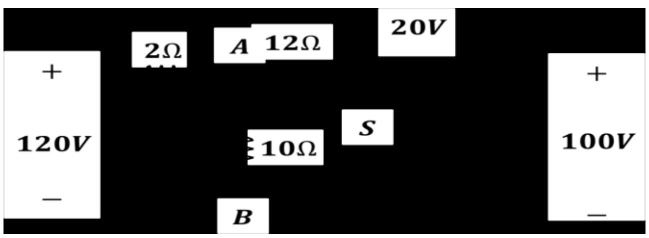
$10I_1 - 11I_2 = 110 \quad (2)$

[redacted]

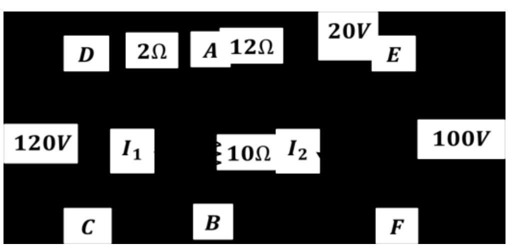
$66.2\,V$

Problem 15:

[redacted]

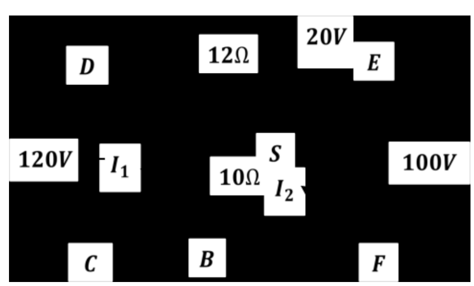


Solution : Case (a): Switch S is open: As no current flows through the switch when it is open, its presence in the circuit can be ignored. The resulting circuit is shown in figure:



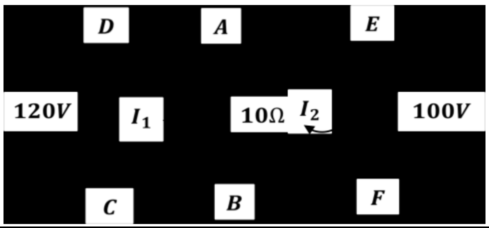
[redacted]

[redacted]



Case (b): Switch S closed

[redacted]



[redacted]

[redacted]

$-12I_1 + 10I_2 = -120 \qquad (1)$

$+10I_1 - 10I_2 = 100 \qquad (2)$

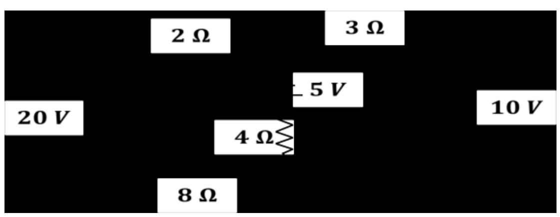
$I_1 = 10\ A\ and\ I_2 = 0\ A$

$I_{AB} = (10 - 0) = 10\ A$

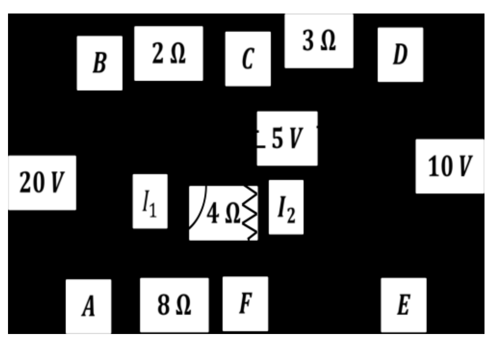
$100\ V$

Problem 16:

Examine whether the power supplied by the source is same as the power dissipated in the circuit shown.



Solution: **Note: To examine whether the power supplied by the source is same as the power dissipated in the circuit, the power dissipated by the resistors and the power supplied or received by the batteries are to be found**



[redacted]

$-14I_1 + 4I_2 = -25 \quad (1)$

[redacted]

$4I_1 - 7I_2 = -5$ (2)

$I_1 = 2.38\,A$ $and\; I_2 = 2.07\,A$

[redacted]

$P_{20} = 20 \times 2.38 = 47.6\,W$

[redacted] $0.31\,A$ [redacted]

$P_5 = 5 \times 0.31 = 1.55\,W$

[redacted]

$P_{10} = 10 \times 2.07 = 20.7\,W$

[redacted] $69.9\,W$

[redacted] $69.9\,W$ [redacted]

Problem 17:

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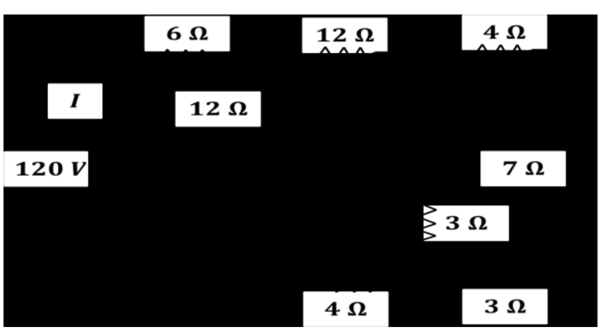

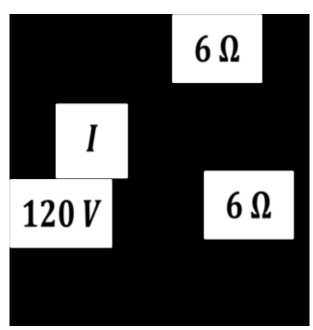

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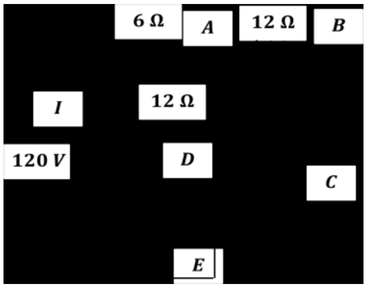
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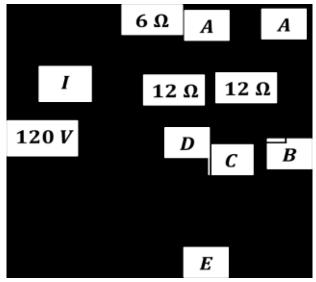
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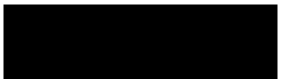

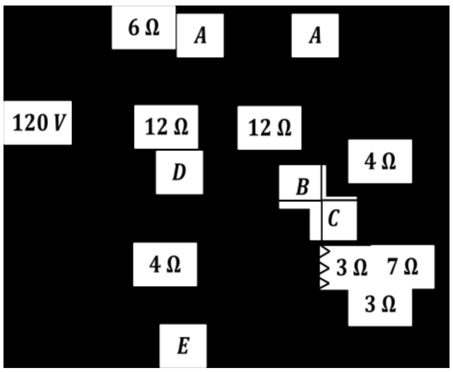
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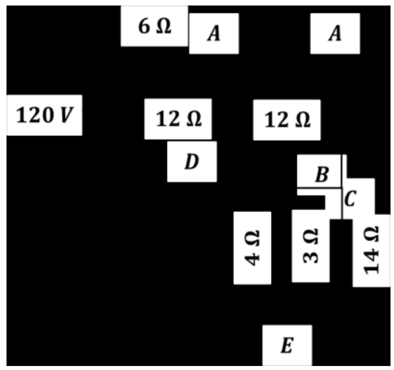
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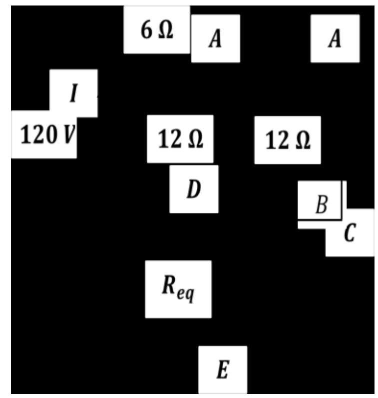
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Module 1

Electromagnetism & Electromagnetic induction

CONTENTS

Electromagnetism:

Review of field around a conductor and coil, magnetic flux and flux density, magneto motive force and magnetic field intensity, reluctance and permeability, definition of magnetic circuit and basic analogy between electric and magnetic circuits. (These topics are not to be considered for setting the examination questions).

Electromagnetic induction:

Definition of Electromagnetic Induction, Faradays Laws, Fleming's right hand rule, Lenz's Law, Statically and dynamically induced e.m.f. Self-inductance, mutual inductance and coefficient of coupling, Energy stored in magnetic field, Illustrative examples. Force on current carrying conductor placed in a magnetic field, Fleming's left hand rule.

Course Outcomes:

After completing this chapter, students should be able to

- **Define magnetic induction.**
- **State the basic law of electromagnetism.**
- **State Lenz's law.**
- **Describe how the right -hand rule for generators can be used to determine the polarity of induced voltage.**
- **Using Faraday's law, calculate the emf across a conductor being passed through a magnetic**

  fi eld.
- **Explain the principles of inductance**
- **Define mutual inductance.**

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Magnetic Fields

The figure shows a Permanent bar a magnet with its north and south poles.
The region, where another magnetic pole, metal or moving electrons experiences a force is called the magnetic field.
Lines of force or flux (Traced, as per the trajectory of the sprinkled iron filings)

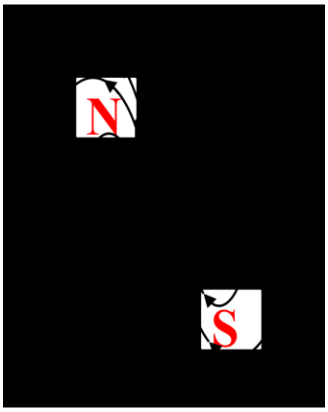


Fig.1. Lines of force around a bar magnet

Magnetic field may be represented by continuous lines of force called magnetic flux. It is generally symbolized by Φ (Phi). The unit of flux is Weber (Wb). By convention, the flux is assumed to leave the North Pole and enter the South Pole, returning to the North pole through the magnet.

ELECTROMAGNETISM:

An electromagnetic field is a magnetic field generated by current flow in a conductor.
Whenever current flows a magnetic field exists around the conductor. A definite relationship exists between the direction of current flow and the direction of the magnetic field. The Right-hand rule is used to determine the direction of flux produced.

RIGHT-HAND OR RIGHT-HAND GRIP RULE

If a current-carrying conductor is grasped with the right hand with the thumb pointing in the direction of current flow, the fingers will point in the direction of the magnetic lines of flux. This is shown in Fi.2. and fig.3.

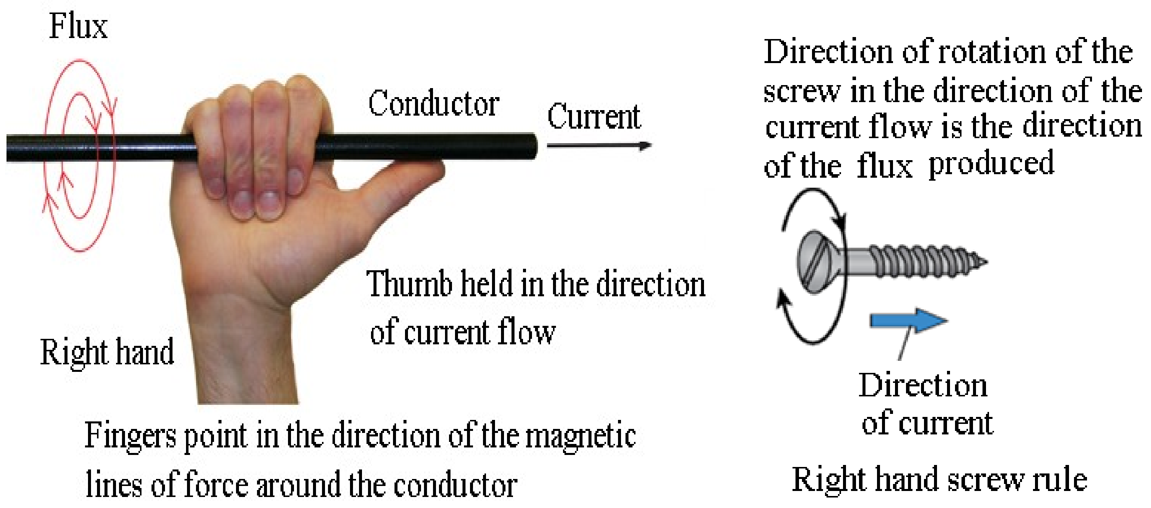


Fig.2. Right hand rule

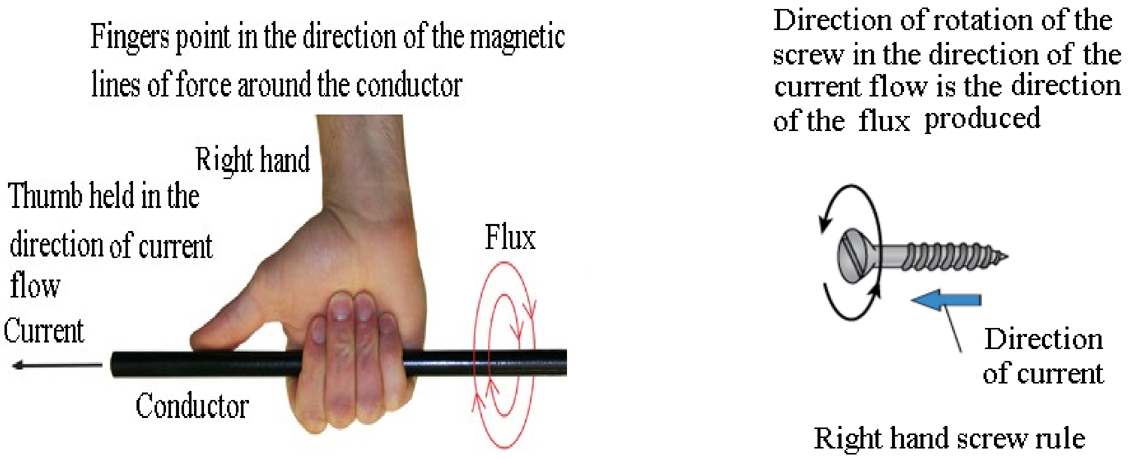


Fig.3. Right hand rule

Representation of magnetic field around a current carrying conductor:

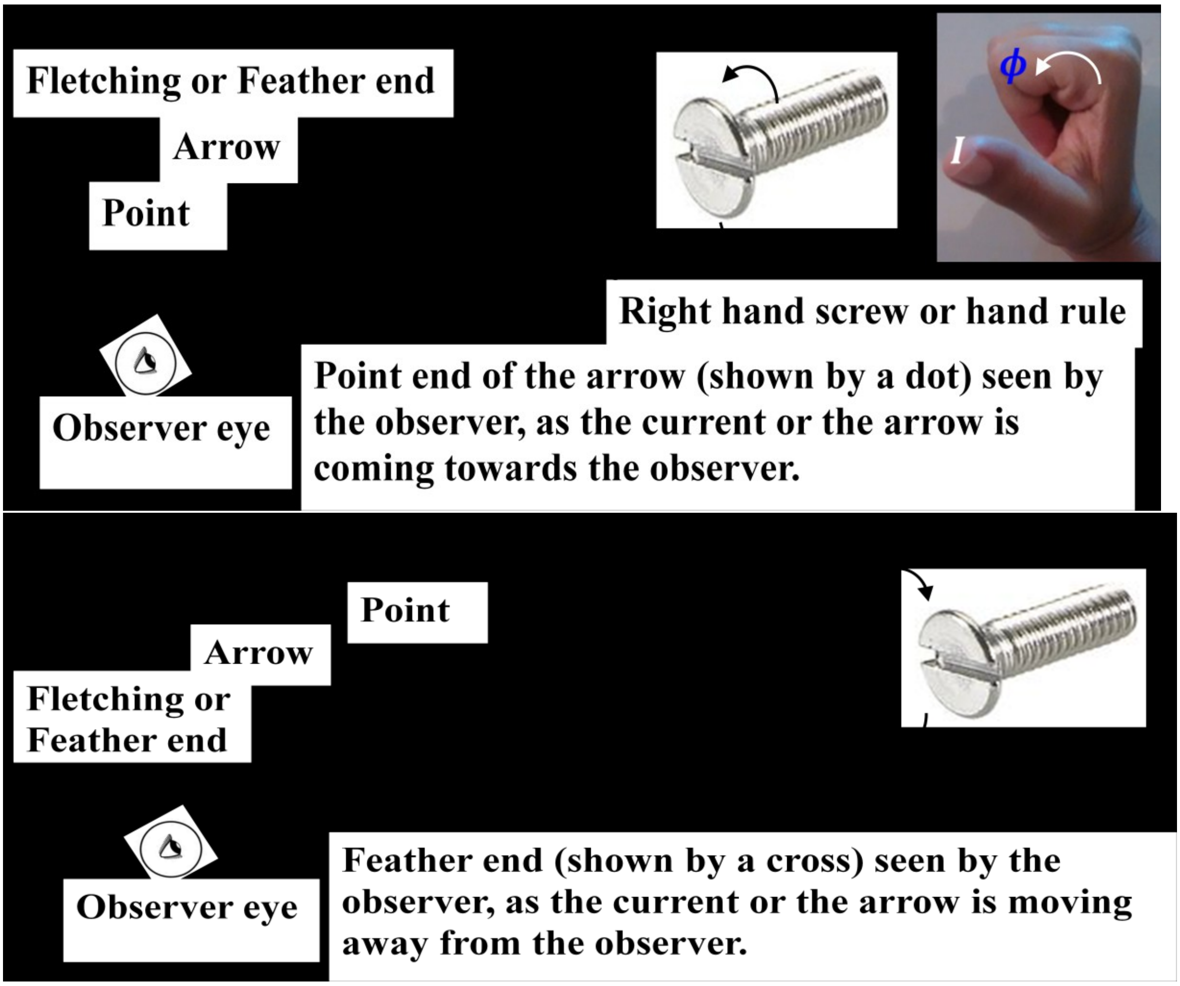


Fig. 4 **magnetic field around a current carrying conductor**

Right hand screw rule for direction of flux: Direction of rotation of the screw to move in the direction of flow of current indicates the direction of flux produced.

Right hand rule for direction of flux: With the right hand thumb in the direction of the current, the curled direction of the fingers indicates the direction of flux produced.

The concentric lines of force of all the loops will be in the same direction through the centre of the loop. This drastically increases the flux and flux density at the centre of the loop or coil, called the core.

The flux can further be increased by arranging the loops close together. In that case, some lines merge and go around combined loops as shown in figure 5.

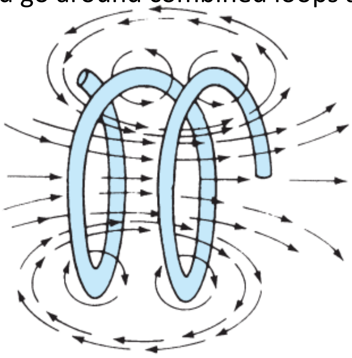


Fig. 5 lines of force due to a loop of conductor**.**

If several turns of insulated wire are formed into a coil, lines of force will enter one end of the coil, pass through it, and emerge at the other end. The lines of force will be completed outside of the coil, as shown in Figure 6.

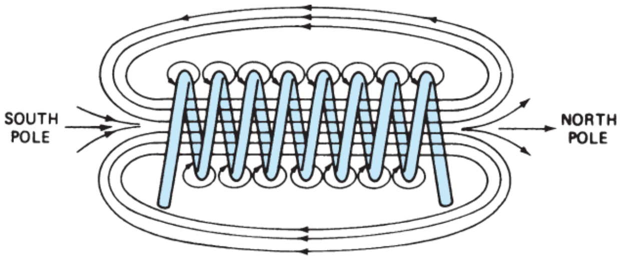


Fig. 6 lines of force due to a coil

The direction of flux set up by the coil is determined by the right hand rule. Refer figure 7.

If the curled fingers of right hand holding the coil are in the direction of the current flow, then the pointing direction of the thumb represents the direction of the flux produced.

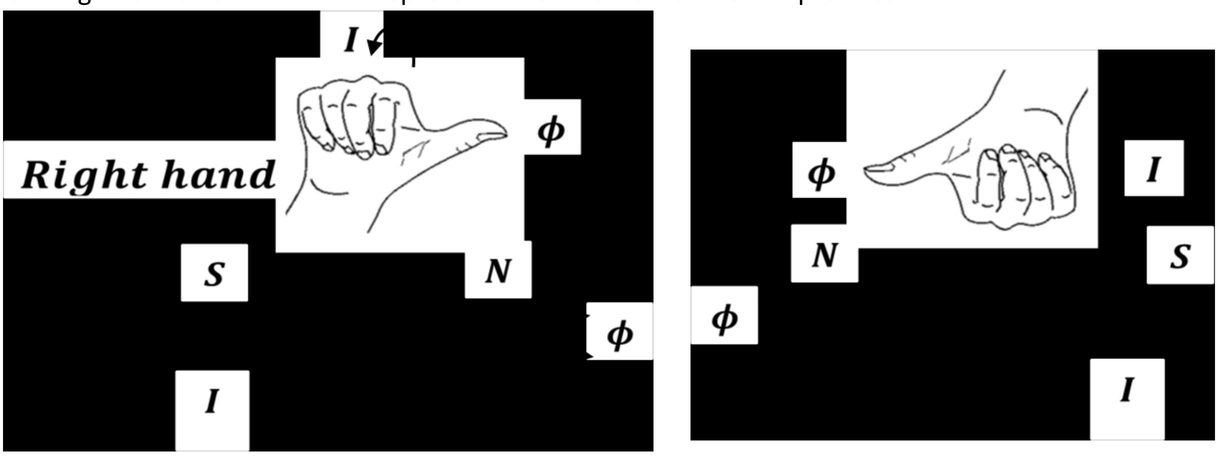


Fig. 7. Right hand rule for coil

Magnetic Strength of Electromagnets: The strength of an electromagnet depends on the Size of the coil, length, material of the core, number of turns in the coil, and amount of current flowing through it. A coil with a magnetic material core is called an electromagnet.

For a given core material and its size, the flux produced depends (neglecting saturation) on the number of turns in the coil and the magnitude of the current. In other words the magnetic field is proportional to ampere turns i.e., ampere I x turns N.
The quantity ampere –turn is called the magneto motive force (mmf).
Though the unit of mmf is ampere, it generally stated as ampere – turns as mmf also depends on the number of turns. Like electromotive force (e.m.f) drives current in an electric circuit, the magneto motive force (mmf) establishes flux and a magnetic field

MAGNETIC FIELD INTENSITY
The strength of the magnetic field set by a coil depends on the value of mmf. The intensity of the magnetic field depends on the length of coil length. Lesser the length of the coil, higher the magnetic field intensity (also called magnetizing force). The intensity is defined as the mmf per unit length and is symbolized by the letter H.
For a solenoid, N = mmf / length = IN /l = B / μ
The field intensity H is at the center of the air core coil. For iron core, H is the intensity through the entire core. The value of H does not depend on the material
RELUCTANCE
While the electric circuit is defined as the path of the electric current, magnetic circuit is defined the path of magnetic flux. Like electric circuit offers resistance to flow of current, the magnetic circuit offers opposition to establishment or flow of flux. The opposition to flux is called reluctance**.**
The reluctance is directly proportional to the length of flux path and inversely proportional to the permeability and to the cross sectional area of the material through which flux is passing. In terms of equation, reluctance $S = l/a\mu_0\mu_r$
Where
L = length of the flux path in meter
A = cross sectional area over which flux is passing
μ_0 = Free space permeability = $4 \pi \times 10^{-7}$ H /m
μ_r =Relative permeability of medium or material through which flux is passing.
PERMEABILITY
Relative permeability is a dimensionless quantity.
Its value can be taken as 1 for vacuum, air, free space and non-magnetic materials.
Its value is greater than one and is high for ferromagnetic materials.
Ferromagnetic materials include iron, steel, nickel, cobalt, and their alloys**.**
Permeability is the measure of the ability of a material to support the formation of magnetic field within itself. Hence, it is the degree of magnetization that a material obtains in response to an applied magnetic field. Magnetic permeability of vacuum μ_0, also called free space permeability, is taken as baseline and is equal to $4\pi \times 10^{-7}$ H/m.
Relative permeability represents the relative ease of establising a magnetic field in a given material.
It is the ratio of fluxdensity B in material caused by some magnetic field H to an induction in vacuum
In the same field. It is dimensionless as it is relative to vacuum permeability.
The ratio Actual flux density within material (B) to magnetic field strength (H) is called the absolute permeability or magnetic permeability μ. The relative permeability μ_r is then defined as the ratio permeability μ of the material to the permeability of the free space μ_0.
If the permeability is very high then the reluctance offered by the flux path will be very small and it can be neglected.

COMPARISON BETWEEN ELECTRIC CIRCUIT AND MAGNETIC CIRCUIT				
Sl No	Electric Circuit		Magnetic Circuit	
	Particulars	Unit	Particulars	Unit
1	Electric circuit is defined as the path of the electric current.		Magnetic circuit is defined as the path of magnetic flux.	
2	Electromotive force, emf	volt	Magnetomotive force, mmf	ampere
3	Resistance R	ohm	Reluctance S	ampere per weber
4	Current I	ampere	Flux Φ	Weber
5	Conductance G	siemen or mho	permeance μ	weber per ampere
6	Current density	A per mm^2	Flux density B	Tesla
7	Electrical insulators are available to insulate live electrical parts.		Though magnetic insulators do not exist, magnetic shields are available.	
8	To maintain a current in a circuit, electrical energy is required.		Once the flux is set up, it does not consume any energy. However, energy is required to maintain the current.	

ELECTROMAGNETIC INDUCTION

Electromagnetic induction refers to the induction (generation) of an e.m.f in a circuit (conductor, turn, loop or coil) due to the changing magnetic flux linked with the circuit. The induced e.m.f in turn, causes a current in the closed circuit or it is said to induce a current. Faraday discovered that,
(1) If the magnetic field through a loop of wire varies in time then an emf is induced around the loop.
(2) An e.m.f is generated when a loop of wire moves from a region of low magnetic field-strength to high magnetic field-strength, and vice versa.
(3) An e.m.f is generated around a loop which rotates in a uniform magnetic field of constant strength.
Faraday's laws: Faraday summarized the above facts and they are known as Faraday's laws of Electromagnetic Induction.

FARADAY'S FIRST LAW: Whenever the flux surrounding a conductor (or through coil, loop, turn or circuit) changes in magnitude, direction or both an e.m.f is induced is in it.

FARADAY'S SECOND LAW: The voltage induced or generated is equal to the time rate of flux linkages or flux cuttings.

The flux encircling a conductor (coil) is said to be linking the conductor. Flux linkage is the product of the number of conductors (number of turns of a coil) and the number of lines of force encircling them.

If N is the number of conductors or turns of the coil and Φ is flux, then the flux linkage $\psi = N\Phi$.

The change in flux linkages, which is same as flux cuttings is $d\psi = N\, d\Phi$.

Mathematically, according to Faraday's second law, induced e.m.f or generated e.m.f

$$e = \frac{d\psi}{dt} = \frac{dN\Phi}{dt} = N\frac{d\Phi}{dt} \; volts$$

POLARITY (DIRECTION)OF INDUCED E.M.F: Fleming's Right hand rule (Empirical) and Lenz's law (based on the conservation of energy) can be used to determine the direction or polarity of the induced e.m.f.

FLEMING'S RIGHT HAND RULE: When the right hand thumb, forefinger and middle finger are held mutually perpendicular to each other, the middle finger shows the direction of the induced e.m.f (or current) if the thumb directed is in the direction of Motion and the Forefinger in the direction of field (Flux).

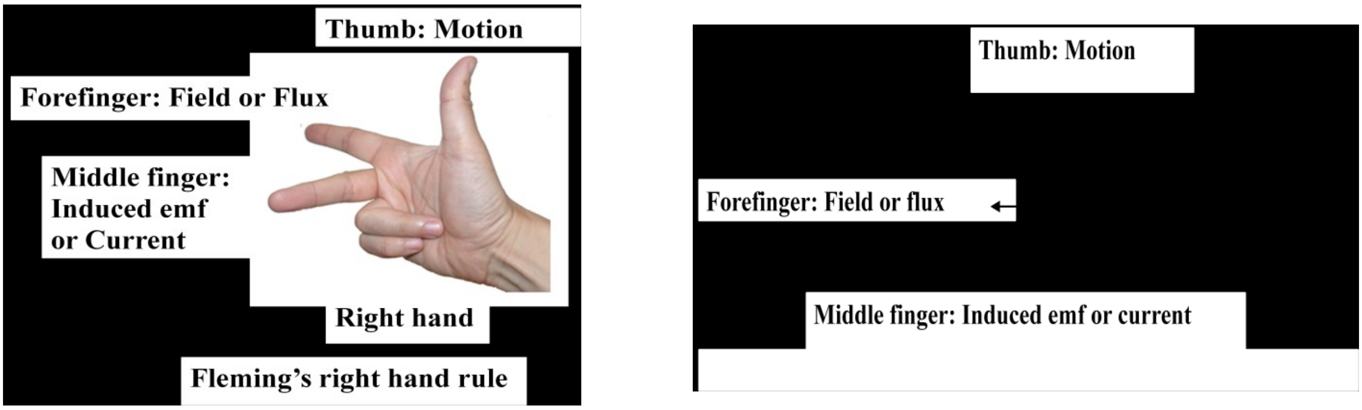


Fig. 9. Graphic representation of Fleming's right hand rule

The Fleming's right hand rule is used to determine the direction of induced e.m.f in generators and direction of back e.m.f in case of motors.

Lenz's law: **Lenz's law states, that the direction of an induced e.m.f is always such that it tends to set up a current to oppose the motion or the change of flux that is responsible for inducing that emf.**

Lenz's law is used to determine the direction of an induced voltage or current.

Consider a coil and a magnet as shown in figure. If the magnet is moved towards the coil, the flux linking the coil increases and therefore an e.m.f is induced according to Faraday's law of electromagnetic induction.

As per Lenz's law, the induced e.m.f should circulate a current in a direction that opposes the cause of the e.m.f i.e., the change in flux linkage.

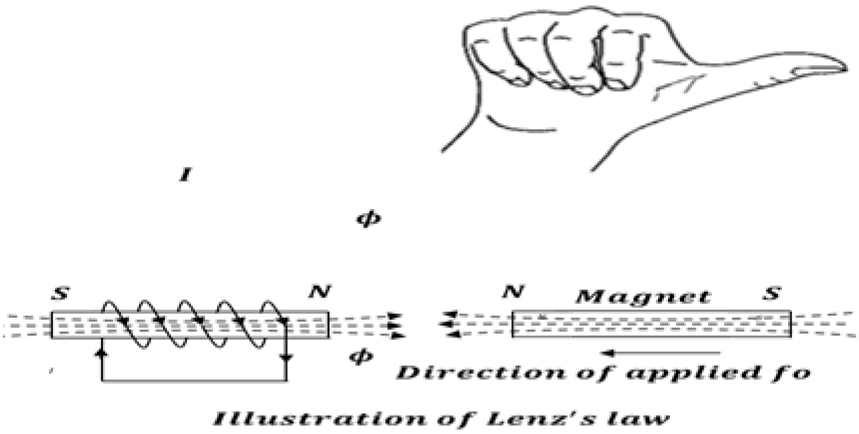


Fig.11. Illustration of Lenz' law

To oppose the change in flux linkage or the approaching North Pole, the e.m.f circulates a current to create a north pole on the right hand side of the coil as shown in figure.

Once the polarity of the poles to be established is determined, using Lenz's law, the direction of the current in the coil can be assigned by the right hand rule.

Since the coil is acting as a source of electricity or generator, the end at which current is coming out of the coil must be the positive polarity of the induced e.m.f.

E.m.f equation considering both Faraday's and Lenz's law :

While the Faraday's law decides the magnitude of the induced e.m.f, the Lenz's law decides the polarity or direction of the e.m.f induced. The induced voltage, which will have a direction to establish currents that oppose the effect which produces them, is indicated by a negative sign.

Hence, as per Faraday's and Lenz's law $e = -N\frac{d\Phi}{dt}\ volts$

TYPES OF INDUCED EMF:

The flux linking the coil or an electric circuit can change when the,

(a) Magnetic field strength changes

(b) Direction of the magnetic field changes

(c) Position of the circuit changes

(d) Shape of the circuit changes

(e) Orientation of the circuit changes.

All the above changes are equivalent as far as the generation of voltage in the circuit is concerned.

Based on the way in which flux linking the circuit changes, the e.m.f induced is classified as dynamically induced e.m.f and statically induced e.m.f.

DYNAMICALLY INDUCED E.M.F: The e.m.f induced in a conductor, coil or circuit when the conductor is moving and field (flux) is stationary or vice versa is called the dynamically induced e.m.f. The relative motion that exists physically between the conductor and the field is cause of change in flux linkage.

In direct current machines, while field remains stationary, the circuit (armature winding) rotates or moves. On the other hand, in case of alternators (alternating current generators) the circuit (armature winding) remains stationary and the field rotates.

STATICALLY INDUCED E.M.F: The e.m.f induced in a conductor or coil when the conductor is stationary and field (flux) is changing with respect to time is called the statically induced e.m.f.

In case of dc, the variation in flux occurs for a short period when the coil (having both resistance and inductance) is energized or de-energized.

However, in case of ac, variation in flux occurs continuously because of change in current with respect to time.

The statically induced e.m.f may be**: self-induced e.m.f or mutual induced e.m.f**.

SELF-INDUCED EMF: The e.m.f induced in a coil because of the time varying flux produced by it and also linking with it is called the self-induced e.m.f.

MUTUALLY INDUCED EMF: The e.m.f induced in a second coil because of the time varying flux produced by the first coil and linking with the second coil or vice versa is called the mutually or mutual induced e.m.f.

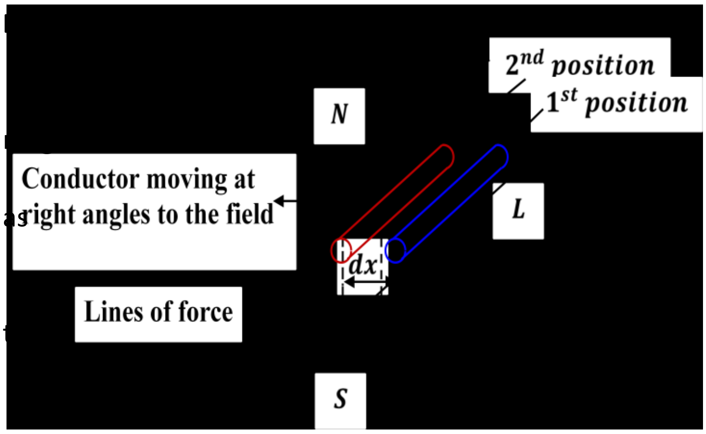
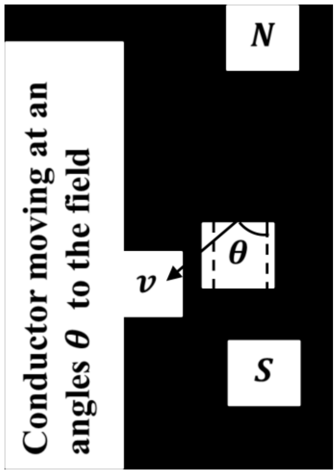


Fig. 12. Dynamic induction

Consider a conductor of length L meter moving at right angles to the field at uniform velocity v m/sec in a field of average flux density B Tesla shown in fig. If the conductor moves through a distance dx in time dt,



Induced e.m.f or generated e.m.f =

$\boldsymbol{e = \frac{dN\Phi}{dt} = \frac{d\Phi}{dt}\ volts}$ **as N = 1**

Change in flux, dΦ = Flux density x area swept by the conductor in time dt

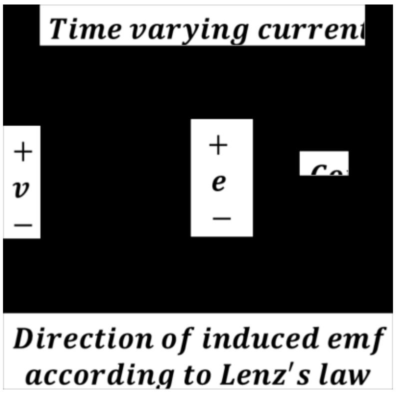
i.e., dΦ = B x dx x L = BLdx

Therefore, $\boldsymbol{e = \frac{d\Phi}{dt} = BLv\ volts}$

When the conductor is moving at an angle to the field, the velocity component perpendicular to the field induces a voltage while the component along the field induces no voltage.

A sine function can be taken as the multiplying factor to determine the induced voltage when the conductor is moving through the field at an angle θ. Therefore, the induced e.m.f in a conductor moving at an angle θ to magnetic field is $\boldsymbol{e = BLv\ Sin\ \theta\ \ volts}$

SELF-INDUCED E.M.F: A magnetic field is formed around any conductor carrying the current. A changing magnetic field linking the conductor or coil induces a voltage in it. The induced voltage is proportional to rate of change of the flux according to Faraday's law.

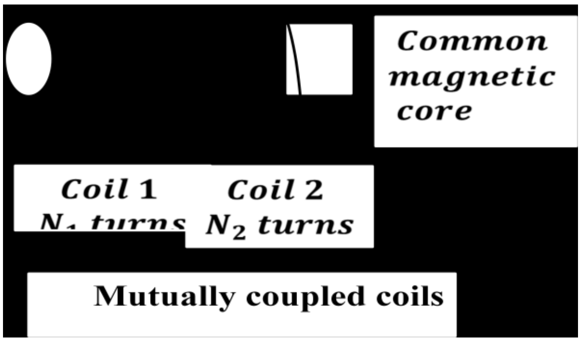


This e.m.f induced is referred to as self-induced e.m.f, counter e.m.f or counter voltage as it opposes the change in flux or the current responsible for it.
According to Faraday's law, self induced e.m.f , $e = N\frac{d\Phi}{dt}$
The direction of the emf is determined by Lenz's law.

The figure shows the applied voltage which is varying with respect to time and the induced e.m.f with polarities at a particular instant.

MUTUALLY INDUCED E.M.F.: Two or more coils which share a common flux are said to be mutually or inductively coupled. If two coils, for example coil 1 and coil 2, are placed close to each other so that the flux of, say coil 1, fully or partially links (cuts across) the coil 2, then every current change in coil 1 will result in a change in the flux linking the coil 2 and an e.m.f in it.

This emf is called the mutually induced emf and is described by Faraday's law. Its direction is determined by Lenz's law.



If the coil 2 is provided a closed path, any change in the current of coil 2 will induce an e.m.f in coil 1. This is also a mutually induced e.m.f. Altogether, with current variations in both 1 and 2 there will be self and mutually induced e.m.f in both the coils.

If all the flux Φ2 produced by the coil 2 links the coil 1 having N1 turns, then according to Faraday's law, Mutualy induced e.m.f in coil 1 due to flux of coil 2 is:

$$e_{M_1} = N_1\frac{d\Phi_2}{dt}$$

If all the flux produced by coil 1 links the coil 2 having N2 turns, then according to Faraday's law, mutually induced e.m.f in coil 2 due to flux of coil 1

$$e_{M_1} = N_2\frac{d\Phi_1}{dt}$$

Self and Mutual Inductances: A circuit in which a change in current causes an e.m.f to be induced within the circuit itself is said to have self inductance, frequently just inductance L. It is also referred to as coefficient of self induction. Inductance is a measure of the amount of flux produced for a given current. That is L = Φ / i

When the conductor is coiled upon itself N number of turns around the same axis, the current required to produce a given amount of flux is reduced by a factor of N compared to a single turn of wire. Thus the inductance of a coil of wire of N turns is L = NΦ/i.

In mutually coupled coils, a changing current in one coil induces a voltage in the neighboring mutually coupled coils. The induced voltage is characterized by mutual inductance which exists between neighboring coils.

The mutual inductance is also called the coefficient of mutual induction**.**

Self-inductance: Inductance is the property of a circuit (coil) that opposes the changes in current. OR Inductance is the property of a circuit (coil) to produce an e.m.f in response to a changing current**.**

According to Faraday's law an e.m.f is induced because of changing current. Therefore

$\boldsymbol{e \propto \frac{di}{dt}}$ **or** $\boldsymbol{e = L\,\frac{di}{dt}}$

The proportionality constant between induced voltage and changing current is defined as the inductance**.**

The direction of the induced e.m.f is determined by Lenz' law and its opposing nature is indicated by a negative sign. Since, the induced e.m.f in a coil of N turns, according to Faraday's law is

$$\boldsymbol{e = N\,\frac{d\Phi}{dt} = L\frac{di}{dt}}$$

$\boldsymbol{L = \frac{Nd\Phi}{di} = \frac{e}{di/dt}}$ **Volt/ampere/second**

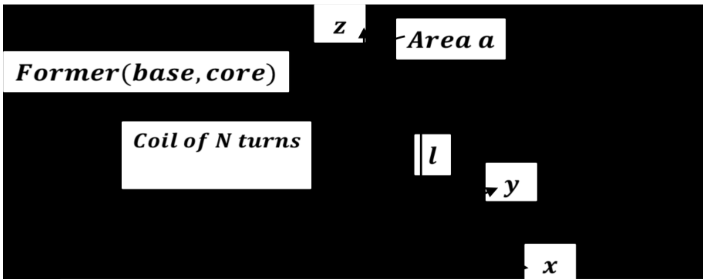
The unit of inductance is henry and is abbreviated by letter H. Because the henry is a large unit, inductors in electrical and electronic circuit typically have inductances measured in milli-henrys (mH), microhenrys(µH), and nanohenrys(nH).

A coil of wire possessing inductance is called an inductor. If the reluctance of the medium remains constant, then the flux linkages varies linearly with the current and therefore $\frac{d\Phi}{di} = \frac{\Phi}{I}$ $or\ L = \frac{N\Phi}{I}$

Therefore, inductance can also be defined as the flux linkage per ampere provided the reluctance of the medium remains constant.

Expression for the inductance in terms of physical dimensions:

Consider a coil having N closely spaced turns wound on a former. If the core has infinite permeability, the magnetic field is confined to the core and has a Z – direction component only.



Coil wound on a former of relative permeability μ_r

For finite permeability much greater than that of the surrounding material, the field is essentially confined to the core and the length of the flux path can be taken as the length of the coili.e., *l.*

If the current in the coil is I, then L = NΦ/I

Since, $\Phi = \frac{mmf\ or\ ampere\ turns}{Reluctance} = \frac{NI}{\frac{l}{\mu_0 \mu_r a}} = \frac{a\mu_0\mu_r NI}{l}$

$$L = \frac{N\ X\ \frac{a\mu_0\mu_r NI}{l}}{I} = \frac{a\mu_0\mu_r N^2}{l} henry$$

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FARADAY'S SECOND LAW: The voltage induced or generated is equal to the time rate of flux linkages or flux cuttings.

The flux encircling a conductor (coil) is said to be linking the conductor. Flux linkage is the product of the number of conductors (number of turns of a coil) and the number of lines of force encircling them.

If N is the number of conductors or turns of the coil and Φ is flux, then the flux linkage $\psi = N\Phi$.

The change in flux linkages, which is same as flux cuttings is $d\psi = N\ d\Phi$.

Mathematically, according to Faraday's second law, induced e.m.f or generated e.m.f

$$\boldsymbol{e = \frac{d\psi}{dt} = \frac{dN\Phi}{dt} = N\frac{d\Phi}{dt}\ volts}$$

ILLUSTRATION OF ELECTROMAGNETIC INDUCTION:

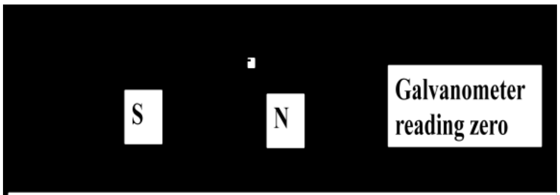
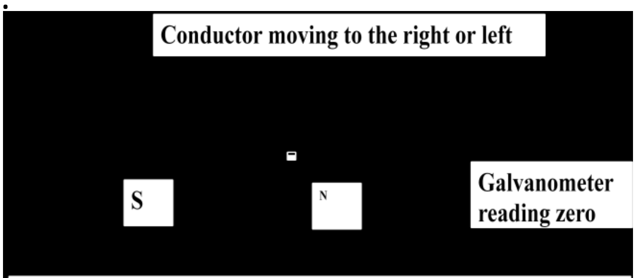
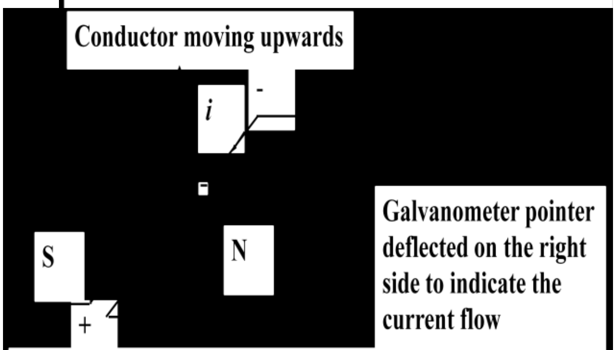


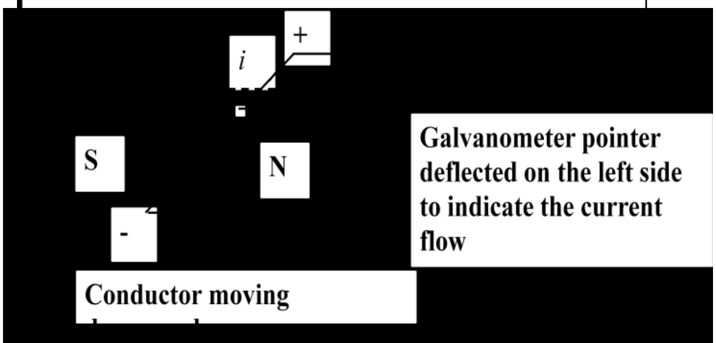
Fig.1a Stationary conductor and field. No emf is induced



Conductor moving parallel to the lines force. No emf is induced



Conductor is moving upwards. Relative motion between the conductor and stationary field induces an emf.

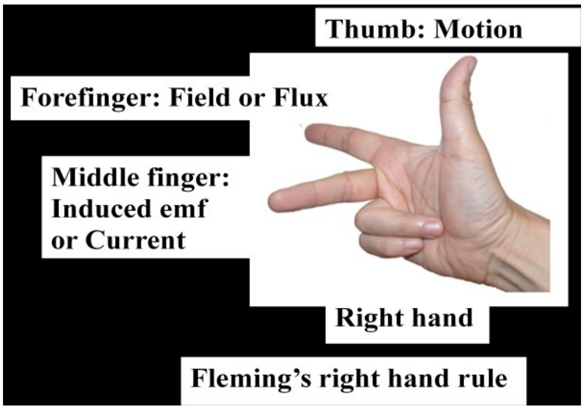


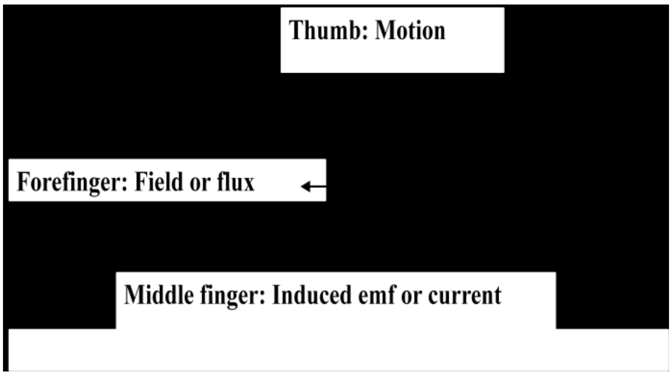
Conductor is moving downwards. Relative motion between the conductor and stationary field induces an emf.

POLARITY (DIRECTION)OF INDUCED E.M.F: Fleming's Right hand rule (Empirical) and Lenz's law (based on the conservation of energy) can be used to determine the direction or polarity of the induced e.m.f.

FLEMING'S RIGHT HAND RULE: When the right hand thumb, forefinger and middle finger are held mutually perpendicular to each other, the middle finger shows the direction of the induced e.m.f (or current) if the thumb directed is in the direction of Motion and the Forefinger in the direction of field (Flux).

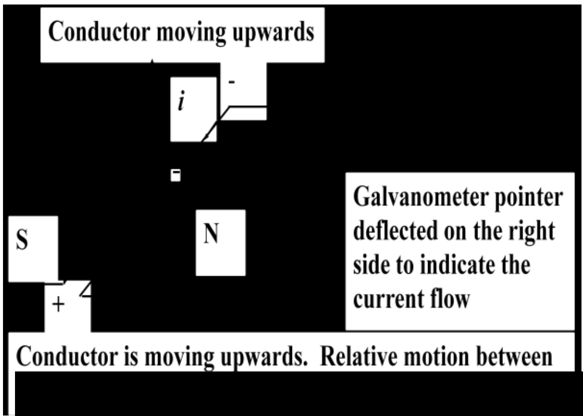
FLEMING'S RIGHT HAND RULE

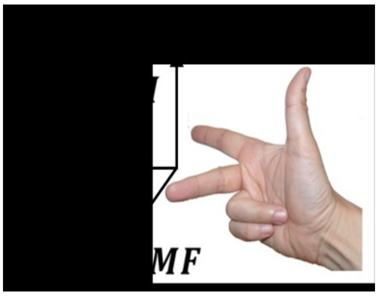


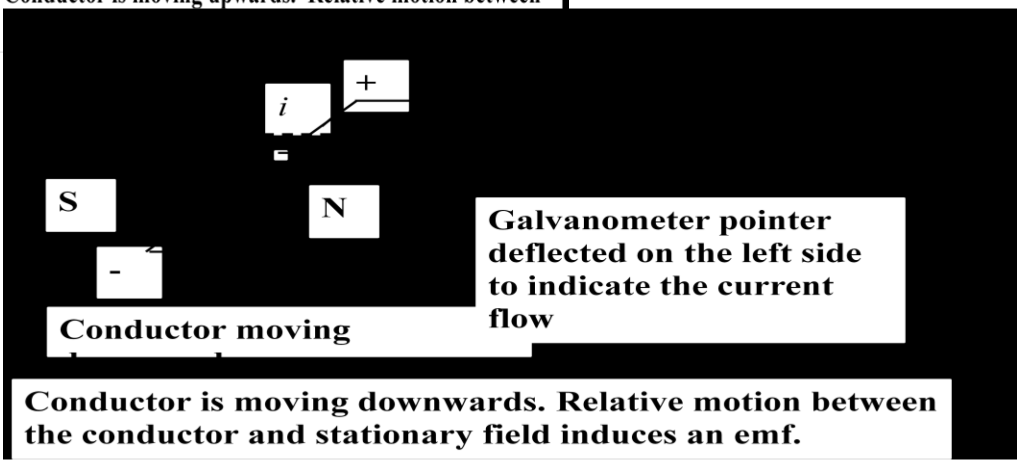


Graphic representation of Fleming's right hand rule

ILLUSTRATION OF ELECTROMAGNETIC INDUCTION:







FLEMING'S RIGHT HAND RULE: The Fleming's right hand rule is used to determine the direction of induced e.m.f in generators and direction of back e.m.f in case of motors.

Lenz's law: **Lenz's law states, that the direction of an induced e.m.f is always such that it tends to set up a current to oppose the motion or the change of flux that is responsible for inducing that emf.**

Lenz's law is used to determine the direction of an induced voltage or current.

Consider a coil and a magnet as shown in figure. If the magnet is moved towards the coil, the flux linking the coil increases and therefore an e.m.f is induced according to Faraday's law of electromagnetic induction.

As per Lenz's law, the induced e.m.f should circulate a current in a direction that opposes the cause of the e.m.f i.e., the change in flux linkage.

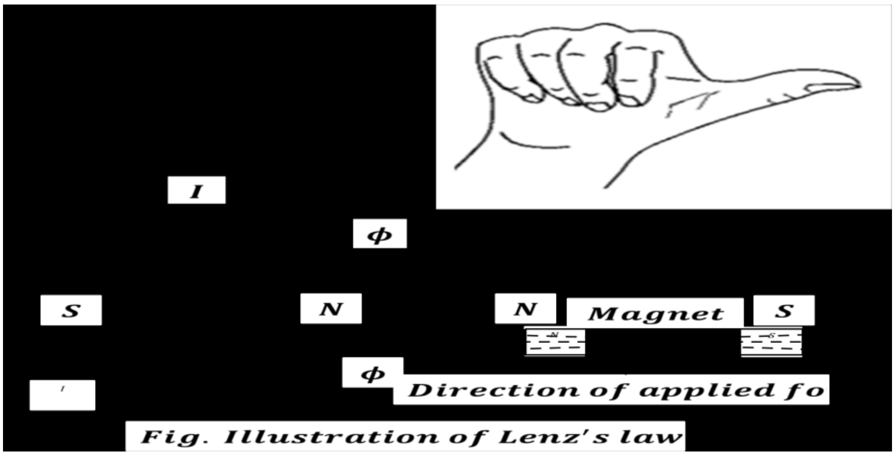


Fig. Illustration of Lenz's law

To oppose the change in flux linkage or the approaching North Pole, the e.m.f circulates a current to create a north pole on the right hand side of the coil as shown in figure.

Once the polarity of the poles to be established is determined, using Lenz's law, the direction of the current in the coil can be assigned by the right hand rule.

Since the coil is acting as a source of electricity or generator, the end at which current is coming out of the coil must be the positive polarity of the induced e.m.f.

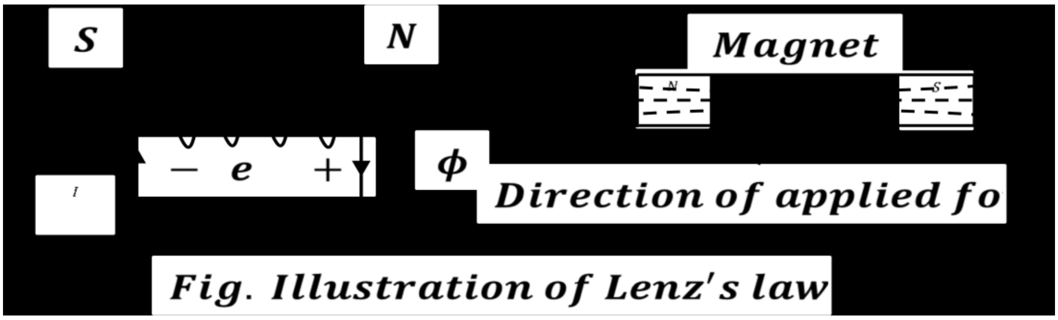


Fig. Illustration of Lenz's law

E.m.f equation considering both Faraday's and Lenz's law :

While the Faraday's law decides the magnitude of the induced e.m.f, the Lenz's law decides the polarity or direction of the e.m.f induced. The induced voltage, which will have a direction to establish currents that oppose the effect which produces them, is indicated by a negative sign.

Hence, as per Faraday's and Lenz's law $e = -N\frac{d\Phi}{dt}\ volts$

TYPES OF INDUCED EMF:

The flux linking the coil or an electric circuit can change when the,

(a) Magnetic field strength changes

(b) Direction of the magnetic field changes

(c) Position of the circuit changes

(d) Shape of the circuit changes

(e) Orientation of the circuit changes.

All the above changes are equivalent as far as the generation of voltage in the circuit is concerned.

Based on the way in which flux linking the circuit changes, the e.m.f induced is classified as dynamically induced e.m.f and statically induced e.m.f.

DYNAMICALLY INDUCED E.M.F: The e.m.f induced in a conductor, coil or circuit when the conductor is moving and field (flux) is stationary or vice versa is called the dynamically induced e.m.f. The relative motion that exists physically between the conductor and the field is cause of change in flux linkage.

In direct current machines, while field remains stationary, the circuit (armature winding) rotates or moves. On the other hand, in case of alternators (alternating current generators) the circuit (armature winding) remains stationary and the field rotates.

STATICALLY INDUCED E.M.F: The e.m.f induced in a conductor or coil when the conductor is stationary and field (flux) is changing with respect to time is called the statically induced e.m.f.

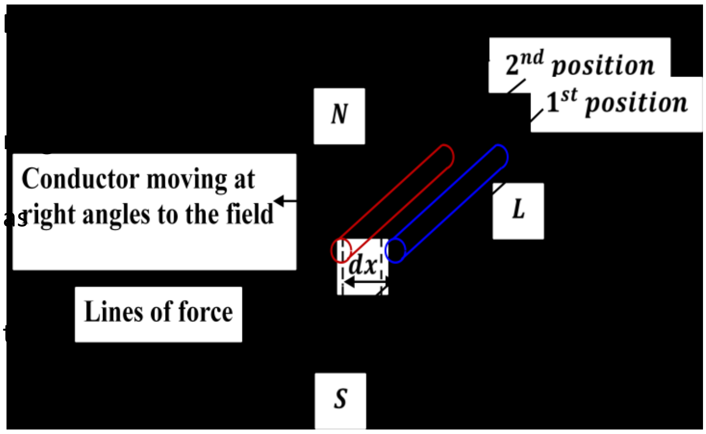
In case of dc, the variation in flux occurs for a short period when the coil (having both resistance and inductance) is energized or de-energized.

However, in case of ac, variation in flux occurs continuously because of change in current with respect to time.

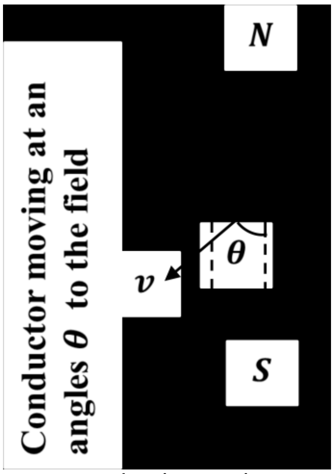
The statically induced e.m.f may be**: self-induced e.m.f or mutual induced e.m.f**.

SELF-INDUCED EMF: The e.m.f induced in a coil because of the time varying flux produced by it and also linking with it is called the self-induced e.m.f.

MUTUALLY INDUCED EMF: The e.m.f induced in a second coil because of the time varying flux produced by the first coil and linking with the second coil or vice versa is called the mutually or mutual induced e.m.f.



Consider a conductor of length L meter moving at right angles to the field at uniform velocity v m/sec in a field of average flux density B Tesla shown in fig. If the conductor moves through a distance dx in time dt, Induced e.m.f or generated e.m.f = $\boldsymbol{e = \frac{dN\Phi}{dt} = \frac{d\Phi}{dt}\ volts}$ **as N = 1**



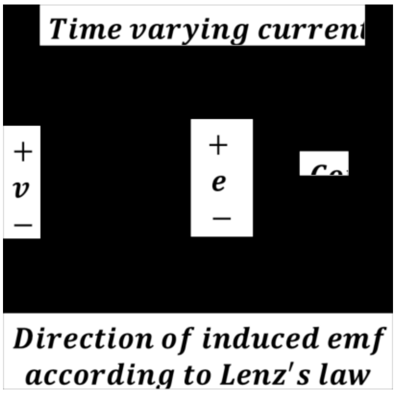
Change in flux, dΦ = Flux density x area swept by the conductor in time dt

i.e., dΦ = B x dx x L = BLdx

Therefore, $\boldsymbol{e = \frac{d\Phi}{dt} = BLv\ volts}$

When the conductor is moving at an angle to the field, the velocity component perpendicular to the field induces a voltage while the component along the field induces no voltage.
A sine function can be taken as the multiplying factor to determine the induced voltage when the conductor is moving through the field at an angle θ. Therefore, the induced e.m.f in a conductor moving at an angle θ to magnetic field is $\boldsymbol{e = BLv\ Sin\ \theta\ \ volts}$

SELF-INDUCED E.M.F: A magnetic field is formed around any conductor carrying the current. A changing magnetic field linking the conductor or coil induces a voltage in it. The induced voltage is proportional to rate of change of the flux according to Faraday's law.

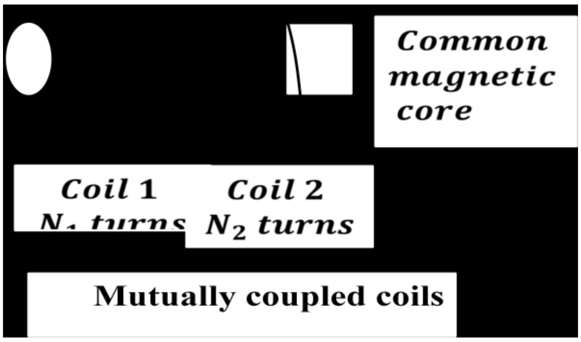


This e.m.f induced is referred to as self-induced e.m.f, counter e.m.f or counter voltage as it opposes the change in flux or the current responsible for it.
According to Faraday's law, self induced e.m.f , $\boldsymbol{e = N\frac{d\Phi}{dt}}$
The direction of the emf is determined by Lenz's law.

The figure shows the applied voltage which is varying with respect to time and the induced e.m.f with polarities at a particular instant.

MUTUALLY INDUCED E.M.F.: Two or more coils which share a common flux are said to be mutually or inductively coupled. If two coils, for example coil 1 and coil 2, are placed close to each other so that the flux of, say coil 1, fully or partially links (cuts across) the coil 2, then every current change in coil 1 will result in a change in the flux linking the coil 2 and an e.m.f in it**.**

This emf is called the mutually induced emf and is described by Faraday's law. Its direction is determined by Lenz's law.



If the coil 2 is provided a closed path, any change in the current of coil 2 will induce an e.m.f in coil 1. This is also a mutually induced e.m.f. Altogether, with current variations in both 1 and 2 there will be self and mutually induced e.m.f in both the coils.

If all the flux Φ2 produced by the coil 2 links the coil 1 having N1 turns, then according to Faraday's law, Mutualy induced e.m.f in coil 1 due to flux of coil 2 is:

$$e_{M_1} = N_1 \frac{d\Phi_2}{dt}$$

If all the flux produced by coil 1 links the coil 2 having N2 turns, then according to Faraday's law, mutually induced e.m.f in coil 2 due to flux of coil 1

$$e_{M_1} = N_2 \frac{d\Phi_1}{dt}$$

Self and Mutual Inductances: A circuit in which a change in current causes an e.m.f to be induced within the circuit itself is said to have self inductance, frequently just inductance L. It is also referred to as coefficient of self induction. Inductance is a measure of the amount of flux produced for a given current. That is L = Φ / i

When the conductor is coiled upon itself N number of turns around the same axis, the current required to produce a given amount of flux is reduced by a factor of N compared to a single turn of wire. Thus the inductance of a coil of wire of N turns is L = NΦ/i.

In mutually coupled coils, a changing current in one coil induces a voltage in the neighboring mutually coupled coils. The induced voltage is characterized by mutual inductance which exists between neighboring coils.

The mutual inductance is also called the coefficient of mutual induction.

Self-inductance: Inductance is the property of a circuit (coil) that opposes the changes in current. OR

Inductance is the property of a circuit (coil) to produce an e.m.f in response to a changing current.

According to Faraday's law an e.m.f is induced because of changing current. Therefore

$e \propto \frac{di}{dt}$ **or** $e = L \frac{di}{dt}$

The proportionality constant between induced voltage and changing current is defined as the inductance.

The direction of the induced e.m.f is determined by Lenz' law and its opposing nature is indicated by a negative sign. Since, the induced e.m.f in a coil of N turns, according to Faraday's law is

$$e = N \frac{d\Phi}{dt} = L \frac{di}{dt}$$

$L = \frac{Nd\Phi}{di} = \frac{e}{di/dt}$ **Volt/ampere/second**

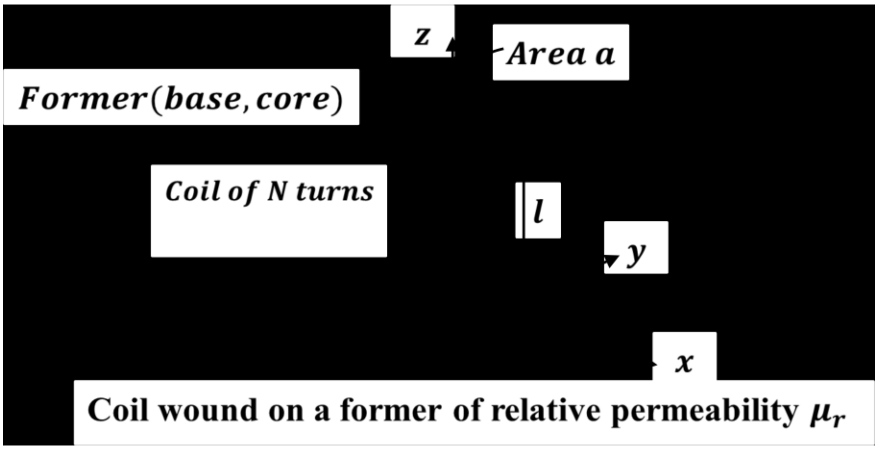
The unit of inductance is henry and is abbreviated by letter H. Because the henry is a large unit, inductors in electrical and electronic circuit typically have inductances measured in milli-henrys (mH), microhenrys(μH), and nanohenrys(nH).

A coil of wire possessing inductance is called an inductor. If the reluctance of the medium remains constant, then the flux linkages varies linearly with the current and therefore $\frac{d\Phi}{di} = \frac{\Phi}{I}$ $or\ L = \frac{N\Phi}{I}$

Therefore, inductance can also be defined as the flux linkage per ampere provided the reluctance of the medium remains constant.

Expression for the inductance in terms of physical dimensions:

Consider a coil having N closely spaced turns wound on a former. If the core has infinite permeability, the magnetic field is confined to the core and has a Z – direction component only.



Coil wound on a former of relative permeability μ_r

For finite permeability much greater than that of the surrounding material, the field is essentially confined to the core and the length of the flux path can be taken as the length of the coili.e., *l.*

If the current in the coil is I, then L = NΦ/I

Since, $\Phi = \frac{mmf\ or\ ampere\ turns}{Reluctance} = \frac{NI}{\frac{l}{\mu_0 \mu_r a}} = \frac{a\mu_0\mu_r NI}{l}$

$$L = \frac{N\ X\ \frac{a\mu_0\mu_r NI}{l}}{I} = \frac{a\mu_0\mu_r N^2}{l} henry$$

Mutual Inductance

Mutual inductance can be defined as the common property of two or more coils whereby an emf is induced in one coil by a current change in the other coil. If all the flux produced by the first coil links the second coil, then the two coils are considered to be tightly coupled. However, in practice all the flux produced by one coil does not link the other.

This leads to a leakage flux that weakens the magnetic coupling. The degree of coupling between two coils is generally quantified by the coefficient of coupling k. The coefficient of coupling or coupling factor is defined as the ratio of the flux linking the two coils to the flux produced by the first or second coil.

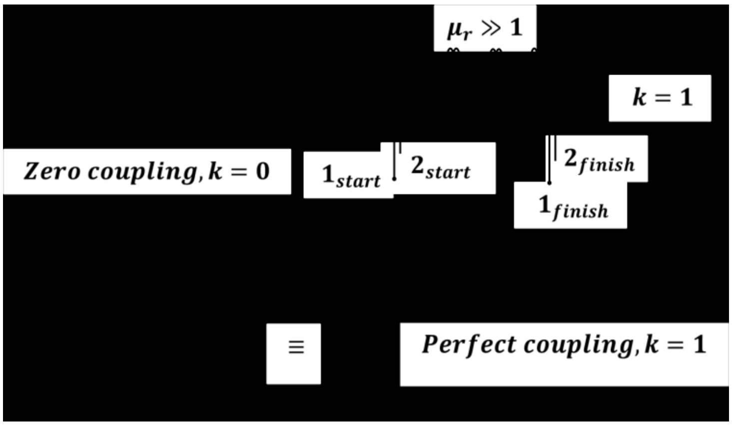
Thus, k = Flux linking both coils / flux produced by the first or second coil

$= \Phi_{12} / \Phi_1 = \Phi_{21} / \Phi_2 \leq 1.0$

Where Φ_{12} is the flux of coil 1 that is linking with both coils. Similarly, Φ_{21} is the flux of coil 2 that is linking with both the coils. If $\Phi_{12} = \Phi_{21} = \Phi_m$, k = 1.0.

The coefficient of coupling is a decimal indicating the percentage of lines of force, with reference the total flux, passing through the other coil. For tight coupling the value of k is one and zero when the coil fluxes are not linking each other.

The two extremities is illustrated in figure. If the coils are well apart, only a small fraction of the flux is linked with the other, and the coils are said to be loosely coupled. A coefficient coupling of 80% is same as 0.8. The decimal form is used in calculations.



Mathematical expression for the mutual inductance:

Consider two coils. Let the number of turns of the two coils, namely, coil 1 and coil 2 be N1 and N2 respectively. Consider the coil 1 is open and the coil 2 is connected to a time varying source. If i2 is the time varying current in coil2, then the mutually induced e.m.f in coil1 due to a changing current in coil 2 is

$$e_{m_{12}} \propto \frac{di_2}{dt} \ or \ e_{m_{12}} = M_{12}\frac{di_2}{dt} \dots\dots\dots\dots (1)$$

If all the flux Φ2 produced by the coil 2, links the coil 1 then

$$e_{m_{12}} = N_1\frac{d\Phi_2}{dt} \dots\dots\dots\dots (2)$$

From equations (1) and **(2)**

$e_{m_{12}} = M_{12}\frac{di_2}{dt} = N_1\frac{d\Phi_2}{dt}$ ……… (3)

And $M_{12} = N_1\frac{d\Phi_2}{di_2}$ ………. (3)

Where

M_{12} is the mutual inductance between the coils 1 and 2.

Similarly, the mutually induced e.m.f in the open circuited coil 2 due the changing current i_1 in coil 1,

$e_{m_{21}} = M_{21}\frac{di_1}{dt} = N_2\frac{d\Phi_1}{dt}\ and\ M_{21} = N_2\frac{d\Phi_1}{di_1}$ ….. (4)

Where M21 is the mutual inductance between the coils 2 and 1.

If the flux produced by the coils are proportional to their respective currents, then

$\frac{d\Phi_1}{di_1} = \frac{\Phi_1}{i_1}\ and\ \frac{d\Phi_2}{di_2} = \frac{\Phi_1}{i_2}$ ……. (5)

With reference to equation (5), equations (3) and (4) can be written as

$$M_{12} = N_1 \frac{d\Phi_2}{di_2} = \frac{N_1 \Phi_2}{i_2} = \frac{N_1}{i_2} X \frac{N_2 i_2}{S_2} = \frac{N_1 N_2}{S_2} \ldots\ldots\ldots\ldots (6)$$

$$M_{21} = N_2 \frac{d\Phi_1}{di_1} = \frac{N_2 \Phi_1}{i_1} = \frac{N_2}{i_1} X \frac{N_1 i_1}{S_1} = \frac{N_1 N_2}{S_1} \ldots\ldots\ldots\ldots (7)$$

Where S1 and S2 are the reluctances offered by the magnetic circuit to fluxes Φ1 and Φ2 respectively. Since the magnetic circuit is common for both the fluxes, S1 = S2 = S. Therforefrom equations (6) and (7)

$$M_{12} = M_{21} = \frac{N_1 N_2}{S} \ldots\ldots\ldots. (8)$$

It is clear from equation (8), that the mutual inductance between the coils is same. Thus the mutual inductance

$$M = M_{12} = M_{21} = \frac{N_1 N_2}{S}$$

The same is true if only a portion of the flux produced by one coil links the other. The unit of mutual inductance is Henry.

RELATION BETWEEN SELF AND MUTUAL INDUCTANCE

Case (a). When all the flux set up by one coil links with the other coil.

Consider two coils having the same magnetic circuit. Therefore all the flux produced by one coil can be assumed linking the other coil. Let the number of turns of the two coils be N_1 and N_2 respectively.

If i_1 and i_2 are the time varying currents in coils I and 2 and Φ_1 and Φ_2 are the fluxes set up by the coils 1 and 2 respectively, then,

$M = N_1 \frac{d\Phi_2}{di_2}$ and $M = N_2 \frac{d\Phi_1}{di_1}$

$$M^2 = N_1 \frac{d\Phi_2}{di_2} \text{ X } \mathrm{N}_2 \frac{d\Phi_1}{di_1} = N_1 \frac{d\Phi_1}{di_1} \text{ X } \mathrm{N}_2 \frac{d\Phi_2}{di_2} = L_1 L_2$$

$$\mathrm{M} = \sqrt{\mathrm{L}_1 \mathrm{L}_2}$$

The mutual; inductance is the geometric mean of the self inductances of the cols.

Case (b): When only a part of the flux set by one coil links with the other coil

Consider two coils on magnetic circuits having different reluctances. Let the number of turns of the two coils be N_1 and N_2 respectively. Let fluxes set up by the coils 1 an2 be Φ_1 and Φ_2 while carrying current i_1 and i_2 respectively.

$M = N_1 \frac{d\Phi_2}{di_2}$ and $M = N_2 \frac{d\Phi_1}{di_1}$

$$M^2 = N_1 \frac{d\Phi_2}{di_2} \text{ X } \mathrm{N}_2 \frac{d\Phi_1}{di_1} = N_1 \frac{d\Phi_1}{di_1} \text{ X } \mathrm{N}_2 \frac{d\Phi_2}{di_2} = L_1 L_2$$

$$\mathrm{M} = \sqrt{\mathrm{L}_1 \mathrm{L}_2}$$

The mutual; inductance is the geometric mean of the self inductances of the cols.

When there is a flux leakage, all the flux of one coil may not link the other. Let $k_1\ \Phi_1$ be the flux of coil 1 linking with coil 2 and $k_2\Phi_2$ be the flux of coil 2 linking coil 1, where k_1 and k_2 are the fractions. As the reluctance for $k_1\Phi_1$ and $k_2\Phi_2$ is same,

$M = N_1 \frac{dK_2\Phi_2}{di_2}$ and $M = N_2 \frac{dk_1\Phi_1}{di_1}$

$$M^2 = N_1 \frac{dk_2\Phi_2}{di_2}\ \mathrm{X}\ \mathrm{N}_2 \frac{dk_1\Phi_1}{di_1} = k_1 k_2 N_1 \frac{d\Phi_1}{di_1}\ \mathrm{X}\ \mathrm{N}_2 \frac{d\Phi_2}{di_2} = k_1 k_2 L_1 L_2$$

$\mathrm{M} = \mathrm{k}\sqrt{\mathrm{L}_1\mathrm{L}_2} = \sqrt{\frac{N_1^2}{S_1}\frac{N_2^2}{S_2}} = \frac{N_1 N_2}{\sqrt{S_1 S_2}}$

Where coefficient of coupling k = $\sqrt{k_1 k_2}$ indicates the amount of coupling between the coils.

We have k_1 = Flux of coil 1 linking with the coil 2 / Total flux produced by coil 1 < 1.0

K_2 = Flux of coil 2 linking with the coil 1 / Total flux produced by coil 2 < 1.0

Air-core coils wound on same former have values of k approximately equal to 0.05 to 0.3, corresponding to 5 to 30% linkage.

When the coils are wound tightly together and mounted on the same magnetic core of very high permeability, the value of k is very close to 1.0. Hence the leakage flux is almost zero and the total flux produced by each of the coils is contained in the core and therefore substantially links both coils.

Application of Electromagnetic induction

1. Generators: Converts mechanical energy to electrical energy.

2. Transformers: Transfers energy from one circuit to another.

Field: Transmission of power over long distances at high voltage, distribution of power at low voltages, measurement of voltage, current etc.

3. Inductors working as filters, energy storage devices as in switched mode power supply, current limiting devices, in electrical transmission system, fluorescent tube circuits to induce a high voltage to cause an arc between electrodes etc.

4. To ignite the mixture of air and petrol in a cylinder of an automobile engine by the use of a spark plug

5. Eddy current which is the result of electromagnetic induction is used

- Tor heating, melting and hardening the metal.
- To detect the defects in a variety of metallic materials.
- Detect whether the metal is heat treated or not as eddy currents are affected by electrical conductivity and magnetic permeability of the material.
- To measure the thickness of the material, non-conducting coatings like paints on conducting material, detect corrosion damage on the skins of the aircraft, erosion and other damages that causes thinning of the material.
- To sort material and to tell if a material is magnetic or non-magnetic.

6. Induction welding is a form of welding that uses electromagnetic induction to heat the work piece.

7. Induction sealing (cap sealing) is a non-contact method of heating a metallic disk to hermetically seal the top of plastic and glass containers.

Field: Pharmaceutical, Food, Dairy, Beverage, Cosmetic etc.

8. Electromagnetic forming (magnet forming) to rapidly reshape metal parts.

9. Tran cranial magnetic simulation (TMS) is used in non-invasive treatment of a host of disorders, including depression.

10.Mineral exploration, Ground water contamination, Salt water intrusion, Mapping geology and soil, Locating buried objects (pipes, barrels, tanks, walls), Archeology, Locating frozen permafrost(a thick subsurface layer of soil that remains below freezing point throughout the year, occurring chiefly in polar regions.), Locating gravel, Locating cavities(caves, abandoned mines) etc.

11. Wireless charging of portable devices such as mobile phones and personal digital assistants.

ENERGY STORED IN AN INDUCTOR:

An increase current in pure inductor from zero to I in t seconds, will be opposed by the induced e.m.f e= L (di/dt). Obviously, work must be done against the induced e.m.f by the source in order to establish the current in the inductor.

Therefore, the energy received during the time dt by the inductor is,

dW = energy supplied or work done by the source = eidt

Total energy received by the inductor in t seconds, $W = \int_0^t eidt = \int_0^t L\frac{di}{dt}idt = \int_0^I Ldi =$ $12\,LI2Joule.$

Force on a current carrying conductor placed in magnetic field:

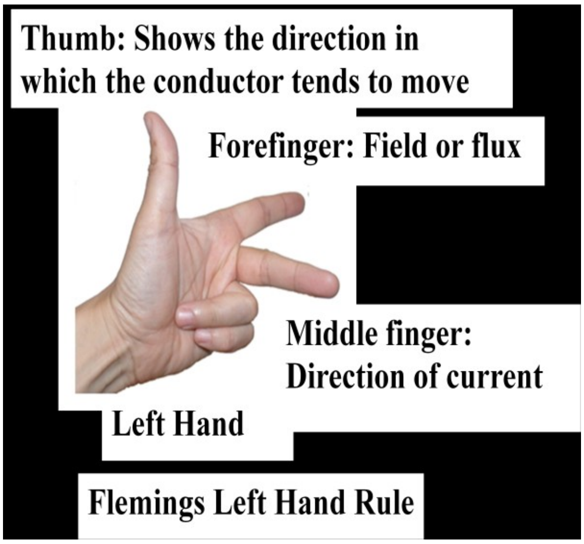
When a current carrying conductor is placed at right angles to the lines of force in a magnetic field, each of the moving charges constituting the current experiences a force called Lorentz force or electromagnetic force according to Lorentz force law.

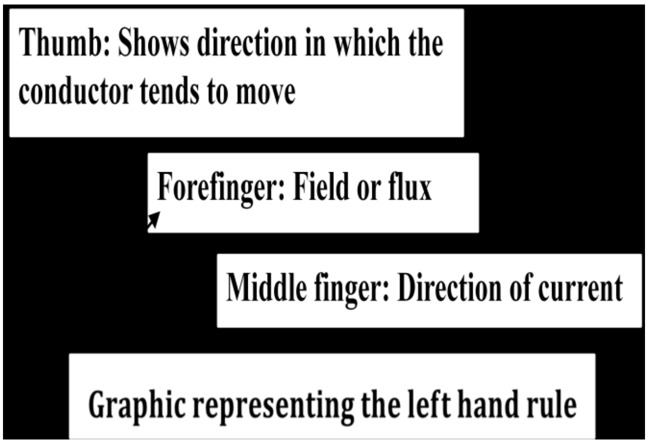
The charges, which cannot escape from the conductor, together create a macroscopic force and transfer this force effect to the conductor. As a result, the whole Lorentz force acts upon the current-carrying conductor.

If the conductor is not mechanically fixed, then it will move according to the acting force.

The force acting on the conductor is proportional to the main field flux density, current flowing through the conductor and the length of conductor within the main field. Therefore, Force on the conductor F = Flux density of the main field x Current flowing through the conductor x Length of the conductor within the main field. i.e., F = BIL Newton.

If the conductor is not perpendicular to the field, but making angle θ with the magnetic field, then
F = BIL sinθ Newton.
The direction of the force can be determined by using Fleming's Left hand rule.





The force developed on a current carrying conductor placed in a magnetic field is utilized to develop torque in electric motors.

Problems and solutions on Electro- magnetic induction

Problem: 1

A conductor, 200 mm long, moves at a constant speed of 2 m/s through a uniform magnetic field of flux density of 1.2 T. What current will flow in the conductor?

(a) If its ends are open-circuited?

(b) If its ends are connected to a 10 ohm resistor?

Solution: (a) e.m.f in the conductor when the direction of motion is perpendicular to the field

$e = BLv \sin\theta = 1.0 \times 0.5 \times 40 \times \sin 90^o = 20$ volts

(b) e.m.f in the conductor when the direction of motion is inclined at 30^o to the direction of field

$e = BLv \sin\theta = 1.0 \times 0.5 \times 40 \times \sin 30^o = 10$ volts

Problem: 2

A rectangular coil has 25 turns and an area of 2.5×10^{-4} m^2. It is placed in a magnetic field of strength of 6.8×10^{-6} T. Calculate the flux linkage when the plane of the coil is (a) parallel to the magnetic field, (b) Perpendicular to the field, (c) State the rate of change of flux linkage in each case.

Solution: (a) When the plane of the coil is parallel to the magnetic field, no flux passes through the coil. Therefore, the flux linkage $\psi = N\Phi = 0$

(b) When the plane of the coil is perpendicular to the magnetic field, flux passes through the coil at right angle to the plane of the coil. Therefore, the flux linkage is maximum.

$\psi = N\Phi = NBA = 25 \times 6.8 \times 10^{-6} \times 2.5 \times 10^{-4} = 42.5 \times 10^{-9}$ Weber.

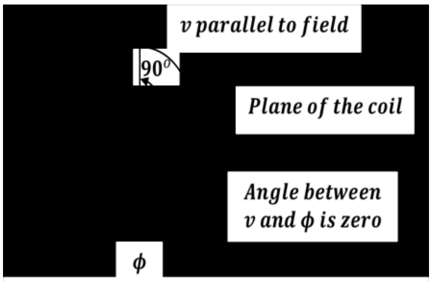
(c) rate of chane of flux linkage is maximum in case (a) and zer in case(b)

Problem: 3

A square coil of 10 cm side and with 100 turns is rotated at uniform speed of 1000 rpm about an axis at right angles to a uniform field having a flux density of 0.5 T. Calculate the instantaneous value of induced e.m.f when the plane of the coil is (a) at right angles to the field, (b) at 30^0 to the field, and (c) in the plane of the field.

Solution: (a) E.m.f induced in the coil e = Blv sinθ. Since the plane of the coil is perpendicular to the field, the angle between the field and the line along which the conductor is moving is zero. That is θ = 0^o

Since θ = 0^o, the induced e.m.f is zero.



Plane of the coil at right angles to the field

(b) Since the plane of the coil is at 30^o to the field, the angle between the field and the line along which the conductor is moving is zero. That is θ = 0^o

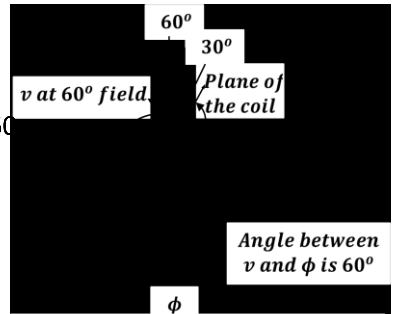
Velocity at which the conductor is moving v

= (π X diameter of the circle created by the coil while rotating X speed in rpm)/ 6

= (π X0.1 X 1000)/ 60 = 5.23 meters/sec

Length of the conductor under the influence of the field L = 0.1 m

E = 2 X 100 X 0.5 X 0.1 X 5.23 X sin 60^o = 45. 3 Volts



Plane of the coil at 30° to the field

(c) (b) Since the plane of the coil is in the plane of the field, the angle between the field and the line along which the conductor is moving is θ = 90^o

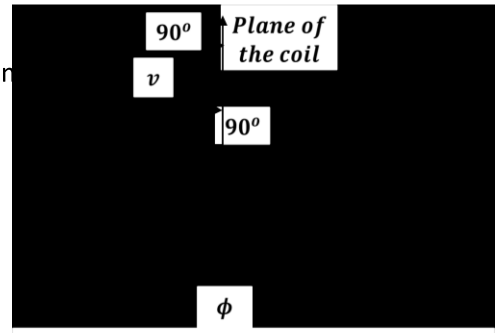
Velocity at which the conductor is moving v

= (π X diameter of the circle created by the coil while rotating X speed in rpn

= (π X0.1 X 1000)/ 60 = 5.23 meters/sec

Length of the conductor under the influence of the field L = 0.1 m

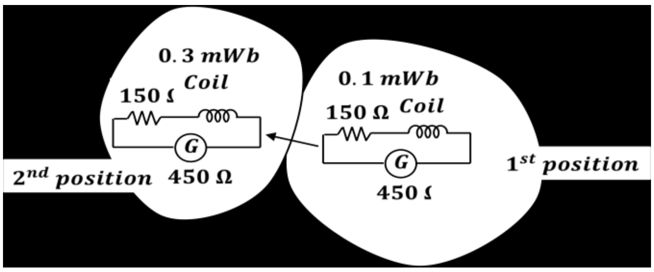
E = 2 X 100 X 0.5 X 0.1 X 5.23 X sin 90^o = 52. 3 Volts



Plane of the coil in the plane of the field

Problem: 4

A coil of resistance 150 Ohm is placed in a magnetic field of 0.1 mWb. The coil has 500 turns and galvanometer of 450 Ohm is connected in series with it. The coil is moved in 0.1 second from the given field to another field of 0.3 mWb. Find the average induced e.m.f and the average current through the coil.



Solution: As the change in the magnetic field cannot be instantaneous at the boundary, the change has to be a gradual one. The field experienced by the coil is not of constant magnitude. Also e.m.f induced is not of constant magnitude. Therefore average e.m.f is to be considered when the coil moves from one field to another.

Average e.m.f induced $e_{av} = N(d\Phi/dt) = 500 \text{ X } [(0.3 – 0.1)\text{x } 10^{-3} / 0.1] = 1.0$ volt
Average current through the coil = e_{av} / (R total) = [1.0 / (150 + 450)] = 1.67 mA.

Problem: 5

The flux Φ linked by the coil of 100 turns varies during the period T of one cycle as follows: from Time t = 0 to t = 0.5 T, $\Phi = \Phi_m(1 – 4t/T)$; and from time t = 0.5 T to t = T , $\Phi = \Phi_m[(4t/T) – 3]$. If T be 0.02 s and Φ_m be 0.02 Wb, calculate the maximum value of e.m.f. Plot flx and e.m.f waves.

Solution:

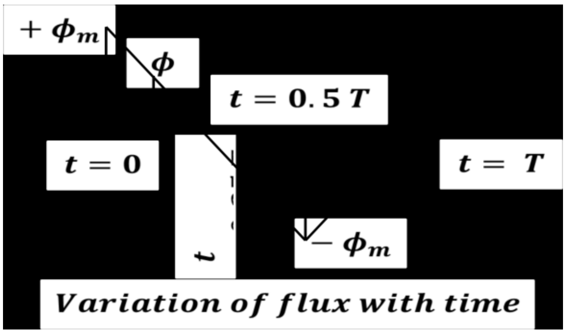
When t = 0, $\boldsymbol{\Phi = \Phi_m\left(1 - \frac{4}{T}t\right) = \Phi_m}$

When t = 0.25T, $\boldsymbol{\Phi = \Phi_m\left(1 - \frac{4}{T}0.25T\right) = 0}$

When t = 0.5T, $\boldsymbol{\Phi = \Phi_m\left(1 - \frac{4}{T}0.5T\right) = -\Phi_m}$

When t = 0.5T, $\boldsymbol{\Phi = \Phi_m\left(\frac{4}{T}t - 3\right) = \Phi_m\left(\frac{4}{T}0.5T - 3\right) = -\Phi_m}$

When t = T, $\boldsymbol{\Phi = \Phi_m\left(\frac{4}{T}t - 3\right) = \Phi_m\left(\frac{4}{T}T - 3\right) = +\Phi_m}$

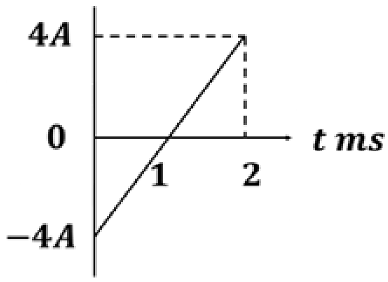


Variation of flux with time

The variation of flux with respect to time i.e., dΦ/dt is linear and its slope of the characteristic.
Therefore, dΦ/dt = Φm / 0.25T = 0.02 / (0.25 x 0.2) = 4
E.m.f induced is constant as dΦ/dt is constant. Maximum value, which is same as that at any instant, is Em = N (dΦ/dt) = 100 x 4 = 400 V.

Problem: 6

The current ramp shown in figure is passed through an 10mH inductor. Find the inductor voltage drop.



Solution:

Inductor voltage drop $e = L (di/dt) = 10 \times 10^{-3} \times [(4 – (- 4))/(2 – 0) \times 10^{-3}] = 40$ Volt

Problem: 7

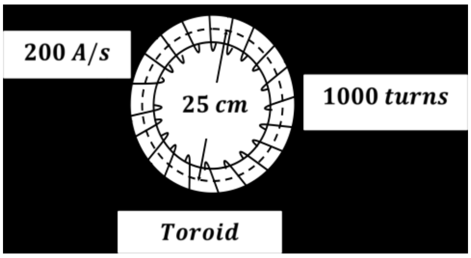
A circuit has 1000 turns enclosing a magnetic circuit of 20 cm^2 in section. With 4 A the flux density is 1 T and with 9 A it is 1.4 T. Find the mean value of the inductance between these currents and the induced emf if the current fell uniformly from 9 A to 4 A in 0.05 second.

Solution:

$L_{average}$ = N(dΦ/dt) = N[(B_2A – B_1A)/(I_2 – I_1)]
= 1000 X20 X 10 – 4 X [(1.4 – 1.0)/(9 – 4)] = 0.16 H
E = $L_{average}$ X (di/dt) = 0.16 X (9 – 4)/ 0.05 = 16 Volts

Problem: 8

Calculate the inductance of a toroid, 25 cm mean diameter and 6.25 cm2 circular cross section, wound uniformly with 1000 turns of wire. Also find the e.m.f induced when a current increasing at the rate of 200 A /s flows in the winding.



Solution:

Inductance $L = \frac{N^2\mu_0\mu_r a}{l}$.

Since the permeability of the core is not given, let it be 1.0

Mean length of flux path in the core or length of the coil, l = π x mean diameter of the coil

= π x 25 = 78.54 cm = 0.7854 m.

$$L = \frac{N^2\mu_0\mu_r a}{l} = \frac{1000^2\ X\ 4\pi X10^{-7}X\ 1.0\ X\ 6.5\ X\ 10^{-4}}{0.7854} = 1.04\ mH$$

E.M.F induced e L di/dt = 1.04 x 10 – 3 X 200 = 0.208 Volts

Problem: 9

An Iron ring 15 cm in diameter and 10 sq.cm in cross section is wound with 200 turns of wire. For a flux density of 1.0 T and a permeability of 500, find the exciting current, the inductance and energy stored.

Solution:

Since, Φ = B X area = mmf / reluctance = (NI)/[l/ a$\mu_0\mu_r$]

Exciting current $I = \frac{Bl}{N\mu_0\mu_r} = \frac{1.0\ X\ \pi\ X0.15}{200\ X\ 4\pi X10^{-7}X500} = 3.75\ A$

Inductance $L = \frac{N^2\mu_0\mu_r a}{l} = \frac{10\ X10^{-4}X\ 4\pi X10^{-7}X500\ X200^2}{\pi X0.15} = 53.3mH$

Alternately, $L = \frac{N\Phi}{I} = \frac{NBa}{I} = \frac{200\ X\ 1.0\ X\ 10\ X\ 10^{-4}}{3.75} = 53.3\ mH$

Energy stored, $W = \frac{1}{2}LI^2 = \frac{1}{2}53.3\ X\ 10^{-3}X3.75^2 = 0.375\ Joules.$

Problem: 10

Two coils having 30 and 600 turns respectively are wound side by side on a closed iron circuit of section 100 cm^2 and mean length 150 cm. Estimate the mutual inductance between the coils if the permeability of iron is 2000. If a current in the first coil grows steadily from zero to 10 A in 0.01 second, find the emf induced in the other coil.

Solution: Since the coils are wound side by side, all the flux produced by one coil can be assumed to link the other. Therefore, coefficient of coupling k = 1

Inductance of 30 turn coil,

$$L_2 = \frac{a\mu_0\mu_r N_1^2}{l} = \frac{100\ X\ 10^{-4} X 4\pi\ X\ 10^{-7} X\ 2000\ X\ 30^2}{1.5} = 0.015\ H$$

Inductance of 600 turn coil,

$$L_2 = \frac{a\mu_0\mu_r N_2^2}{l} = \frac{100\ X\ 10^{-4} X 4\pi\ X\ 10^{-7} X\ 2000\ X\ 600^2}{1.5} = 6.03\ H$$

M = 1.0 √(0.015 X 6.03) = 0.3 H

E.m.f induced in the other coil having 600 turns is :

E2 = M(di_1 /dt) = 0.302 x[(10 – 0)/0.01] = 302 Volt

Problem: 11

Two coils, X of 12000 turns and Y of 15000 turns, lie in parallel planes so that 45 % of the flux produced by X links coil Y. A current of 5 A in X produces 0.05 mWb while the same current in Y produces 0.075 mWb. Calculate the mutual inductance and coefficient of coupling.

SOLUTION:

Mutual inductance $M = \frac{a\mu_0\mu_r N_X N_Y}{l} = \frac{N_X k_Y \emptyset_Y}{I_Y} = \frac{N_Y k_X \emptyset_X}{I_X} = k\sqrt{L_X L_Y}$

$$M = \frac{N_Y k_X \emptyset_X}{I_X} = \frac{15000\ X\ 0.45\ X\ 0.05 X\ 10^{-3}}{5} = 67.5\ mH$$

Inductance of coil X is $L_X = \frac{N_X \emptyset_X}{I_X} = \frac{12000\ X\ 0.05\ X 10^{-3}}{5} = 0.12\ H$

Inductance of coil Y is $L_Y = \frac{N_Y \emptyset_Y}{I_Y} = \frac{15000\ X\ 0.075\ X 10^{-3}}{5} = 0.225\ H$

Coefficient of coupling k = $\frac{M}{\sqrt{L_X L_Y}} = \frac{67.5 X 10^{-3}}{\sqrt{0.12 X 0.225}} = 0.41$

Problem: 12

Two identical 1000 turns coils A and B lie in parallel planes such that 60% of the flux produced by one coil links with the other. A current of 5 A in A produces in it a flux of 0.05 mWb. If the current changes from +6 A to – 6 A in 0.01second, what will be the magnitude of the e.m.f in coil B? Calculate the self inductance of each coil and the mutual inductance.

Solution:

E.m.f induced in the coil B, $e_B = N_B(d\Phi_B/di_A) = M\ (di_A/dt)$

Mutual inductance M = k √($L_A L_B$)

k = √($K_A K_B$)

Since the coils are identical, the percentage of flux of each coil and linking the other is same.

That is $k_A = k_B = 0.6$

Therefore $k = \sqrt{(k_A \times k_B)} = 0.6$

Inductance of each coil LA = LB = NΦ / I = $(1000 \times 0.05 \times 10^{-3}) / 5 = 10$ mH

M = 0.6 x 10 mH = 6 mH

E = 6 x 10 – 3 X [6 – (- 6)] / 0.01 = 7.2 V

Problem: 13
Two toroidal solenoids are wound around the same form so that the magnetic field of one passes through the turns of the other. Solenoid 1 has 700 turns and solenoid 2 has 400 turns. When the current in solenoid 1 is 6.52 A, the average flux through each turn of solenoid 2 is 0.0320 Wb.
(a) What is the mutual inductance of the pair of solenoids?
(b) When the current in solenoid is 3.54 A, What is the average flux through each turn of solenoid 1.

Solution: Since $M = \sqrt{L_1 L_2}$ and $\frac{L_1}{L_2} = \frac{N_1^2}{N_2^2}\ OR\ L_2 = L_1 \frac{N_2^2}{N_1^2}$

$$M = \sqrt{L_1 L_1 \frac{N_2^2}{N_1^2}} = K L_1 \frac{N_2}{N_1}$$

As the magnetic field of one is passing through the turns of the other, K = 1

$L_1 = N_1 \Phi_1 / I_1 = 700 \times 0.032 / 6.52 = 3.44$ H

M = 1.0 x 3.44 x (400/ 700) = 1.97 H

Average flux through each turn of solenoid 1, $\Phi_2 = L_2\ I_2 / N_2$

L2 = M2 /k L1 = 1.97^2 /(1.0 x 1.37) = 1.13 H

Φ_2 = 1.13 x 2.54 / 400 = 7.18 mH

Problem: 14
A pair of poles 5 cm X 3 cm produce a flux of 2.5 mWb. A conductor is placed in this field with its length parallel to the longer dimension of the poles. When a current is passed through the conductor, a force of 1.25 N is exerted on it. Determine the value of the current. If the conductor is now placed at 45^o to the field, what would be the force exerted?
Solution:

Since force on the conductor when it is placed perpendicular to the field is F = BIL,
Current I = F/ BL
Flux density B = Φ / Area = $\frac{2.5\ x\ 10^{-3}}{5\ X\ 3X10^{-4}}$= 1.67 T
Length of the conductor = longer dimension of the poles = 5 cm.
$I = \frac{1.25}{1.67\ x\ 5\ x10^{-2}} \approx 15\ A$
Force experienced by the conductor when placed at 45^o to field and carrying a current of 15 A is
F = BIL Sinθ = 1.67 x 15 x 5 x 10^{-2} x sin45^o = 0.886 N